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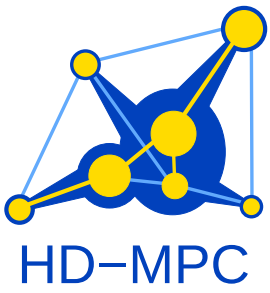

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Table of contents

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Executive Summary

This report describes the research activity in the Seventh Framework Programme, Theme 3 "Information and Communication Technologies", STREP research project **Hierarchical and Distributed Model Predictive Control of Large Scale Systems- HD-MPC**, focusing on WP2 - "Definition of the hierarchical architecture for control design", task 2.1 "Literature survey" and task 2.2 "Methods for system decomposition and hierarchical control architectures".
The report is organized in three main chapters:

- Chapter 1 reviews a number of approaches based on Model Predictive Control for the design of decentralized, distributed and hierarchical control architectures for large scale systems. Starting from the analysis of completely decentralized systems, attention is then focused on distributed control systems, where the local regulators can exchange information to reach a consensus on their actions, or where a high level coordinator is used. The review considers also the main approaches to the design of hierarchical control, used with the aim to control a system described at different levels of accuracy or to cope with an intrinsic hierarchical structure of the plant. Finally, the problem of coordinating a number of dynamically independent systems with coupling constraints is considered. For all the structures reviewed, the underlying rationale, their merits and limitations are discussed, the main references to the literature are reported and some future developments are suggested.

- Chapter 2 presents a number of methods for system partitioning and model reduction. First, the main selection criteria of the input-output pairings suitable for a subsequent design of decentralized control structures are briefly described. They are based on the Relative Gain Array method, and on a number of variations proposed in the literature, as well as on the analysis of the controllability and observability gramians. The final part of the chapter is devoted to summarize the most efficient techniques for model order reduction. This problem is important in the design of hierarchical control structures where the regulators at the higher levels of the structure are designed starting from quite simple models of the plant under control, which typically represent its behavior at low frequency.

- Chapter 3 describes the model and the simulator of a complex chemical benchmark, where the plant is made by three reactors and three distillation columns. The presence of recirculating flows makes the system strongly interacting, so that the design of a decentralized/distributed control system is difficult. In the considered configuration, the overall model of the plant has 183 state variables, 6 inputs and 6 outputs. The model obtained through numerical linearization around an equilibrium is then used to test some of the methods for system partitioning described in the previous chapter. References to the open software code are reported in the Appendix.

Chapter 1

Architectures for distributed and hierarchical control with MPC

Technological and economical reasons motivate the development of process plants, manufacturing systems and traffic networks with an ever increasing complexity. These large scale systems, often composed by many interacting subsystems, can be difficult to control with a centralized control structure due to the required inherent computational complexity, due to robustness and reliability problems and due to communication bandwidth limitations. For all these reasons, many distributed control structures have been developed and applied over the last forty years. Among them, it is worth mentioning completely decentralized structures, distributed control systems with exchange of information between local regulators, and hierarchical structures. Due to the wide range of the problems considered and of the goals to be achieved, it is not always trivial to properly classify all the proposed solutions and to judge their merits and limitations. The aim of this Chapter is to review the main approaches adopted, to propose a classification criterion and to provide a wide list of references focusing the attention on the methods based on the Model Predictive Control (MPC) approach, see e.g. [50]. The reason for this choice is due to the ever increasing popularity of MPC in the process industry and in other fields, such as road traffic control, distribution systems, manufacturing systems, and to its capability to handle static and dynamic constraints on the plant variables. Moreover, in distributed control systems it is easy for any local regulator designed with MPC to predict its future control actions and to transmit them to neighboring local control units. This information is often fundamental to achieve performance comparable to those ideally provided by a centralized control structure.
The Chapter is organized as follows. In Section 1.1 completely decentralized control structures are considered and some fundamental references in the field are reported together with a description of the very few results available concerning decentralized MPC. Section 1.2 describes the main approaches proposed so far to the design of distributed MPC systems, where information is transmitted among local regulators to achieve global stability and performance results. Section 1.3 is devoted to introduce a hierarchical control structure where the action of local (decentralized) regulators is coordinated by an algorithm operating at a higher level. The main ideas underlying the design of this coordinator are summarized together with the approaches adopted in the MPC literature. Section 1.4 deals with hierarchical multilayer systems, i.e. control systems made by a number of control algorithms working at different time scales. Multilayer structures are useful either to control plants characterized by significantly different dynamics or to use different models of the same plant with the aim to optimize a number of criteria. Both these situations are described and the available results are summarized. Distributed MPC algorithms have also been proposed to coordinate totally independent systems in

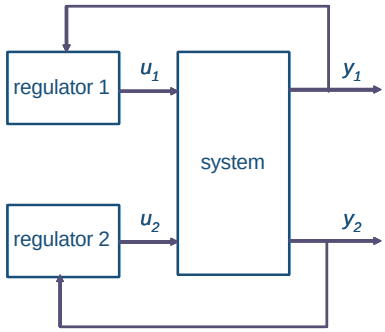

Figure 1.1: Decentralized control of a two input (u_1, u_2) - two output (y_1, y_2) system.

order to achieve a common target and to deal with joint constraints. These cases and the proposed solutions are reviewed in Section 1.5. Finally, Section 1.6 briefly describes some open issues, which are partially considered in the following chapters.

1.1 Decentralized control

In decentralized control, see Figure 1.1, both the control (input u) and the controlled (output y) variables are grouped into disjoint sets. Then, these sets are coupled to produce non overlapping pairs for which local regulators are designed to operate in a completely independent fashion. The design problem is trivial when the interactions (static or dynamic) among the inputs and the outputs of different pairs are weak, while it is well known that strong interactions can even prevent one from achieving stability and/or performance with a decentralized control structure. Classical textbooks dealing with decentralized control are [87], [47], while a milestone paper in the field is [81]. More recently, decentralized control has been considered in the papers [88], [7], [14], which also report an up-to-date list of references.
A decentralized state-feedback MPC algorithm for nonlinear discrete-time systems has been proposed in [52], where closed loop stability is obtained with the inclusion of a contraction constraint in the optimization problem to be solved at any time instant. In [72] a stabilizing state-feedback regulator for nonlinear discrete-time systems is derived by looking at the plant interactions as perturbations (disturbances) to be rejected, by designing robust MPC algorithms [53] and by resorting to Input to State Stability (ISS) concepts, see e.g. [41].

1.2 Distributed control

In distributed control structures, like the one shown in Figure 1.2, it is assumed that some information is transmitted among the local regulators, so that each one of them has some knowledge on the behavior of the others. In distributed MPC algorithms, the information transmitted typically consists of the future predicted control and states computed locally, so that any local regulator can predict the mutual effects of the actions of the others over the considered prediction horizon.

Within the wide set of distributed MPC algorithms proposed in the literature, a further classification can be made depending on the topology of the communication network. Specifically, the

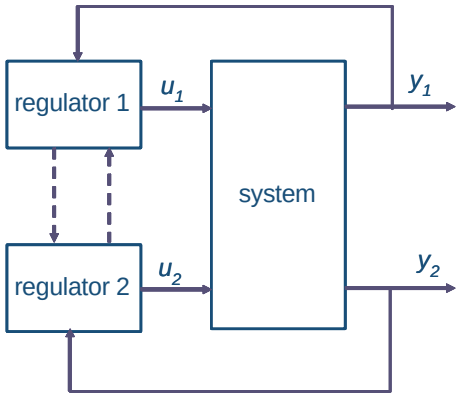

Figure 1.2: Distributed control of a two input (u_1, u_2) - two output (y_1, y_2) system.

following cases can be considered:

- information is transmitted (and received) by any local regulator to all the others (*distributed/fully connected* algorithms);
- information is transmitted (and received) by any local regulator to a given subset of the others (*distributed/partially connected* algorithms).

A distributed/partially connected information structure can be convenient in the case of large scale systems made up by a great number of loosely connected subsystems. In these cases, limiting the information exchange among directly interacting subsystems produces a negligible performance deterioration. An interesting discussion on this point referring to chemical processes is reported in [78]. The exchange of information among local regulators can be made according to different protocols:

- information is transmitted (and received) by the local regulators only once within each sampling time (*distributed/noniterative* algorithms);
- information can be transmitted (and received) by the local regulators many times within the sampling time (*distributed/iterative* algorithms).

It is apparent that the amount of information available to the local regulators with distributed/iterative algorithms is much higher, so that an overall iterative procedure can be set-up to (hopefully) arrive to a global consensus on the actions to be taken within the sampling interval. To this regard however, a further classification has to be considered:

- distributed algorithms where each local regulator minimizes a local performance index (*distributed/independent* algorithms);
- distributed algorithms where each local regulator minimizes a global cost function (*distributed/cooperating* algorithms).

As discussed in [95] (see also [96]) by means of game theory considerations (see [8]), it is apparent that in iterative distributed/independent algorithms each local regulator tends to move towards a Nash equilibrium, while iterative distributed/cooperating methods seek to achieve the Pareto optimal

solution provided by an ideal centralized control structure [1]. However, Nash equilibria can even be unstable and far from the Pareto optimal solution, so that specific constraints have to be included in the MPC problem formulation to guarantee closed-loop stability.

As for the MPC algorithms published in the literature, the state feedback method described in [11] (see also [33]), for discrete-time linear systems belongs to the set of distributed/independent, noniterative algorithms. A stability constraint is also included in the problem formulation, although stability can be verified only a-posteriori with an analysis of the resulting closed-loop dynamics. Nash equilibrium solutions are searched in the distributed/independent, iterative, fully connected methods developed in [17], [45] for discrete-time unconstrained linear systems represented by input-output models. Linear discrete-time systems are also considered in [95], where an iterative, cooperating method with many interesting properties is presented. In particular, the proposed approach guarantees the attainment of the global (Pareto) optimum when the iterative procedure converges, but still ensures closed-loop stability and feasibility if the procedure is stopped at any intermediate iterate.
A partially connected, noniterative, independent distributed MPC algorithm for discrete-time nonlinear systems has been presented in [34]. The approach consists of describing the effect of the interconnections among the subsystems as disturbances acting on the local models. The values of these disturbances can be predicted from the predicted state trajectories broadcasted by the local regulators. Then, a min-max approach aimed at minimizing local cost functions under the worst-case disturbance allows one to compute parametrized distributed control laws. A feasibility property is proven together with convergence to a set. It is believed that the method could be further developed according to many recent results on closed-loop robust MPC, see e.g. [51], [53].
A distributed/independent, noniterative, partially connected MPC algorithm guaranteeing stability for nonlinear continuous-time systems has been presented in [18], where information is only transmitted among neighboring subsystems. The stabilizing properties of the method proposed in [18] heavily rely on the assumption that the mutual dynamic interactions among the subsystems are limited and on a consistency constraint included in the MPC problem formulation forcing the actual input and state sequences to not differ too much from their predicted values. The feasibility and stability proofs are based on the techniques described in [58] and share many ideas with the robust open-loop MPC algorithms developed in [13], [46], [53].
Relying on the methods for distributed state estimation and control presented in [60] and [61], distributed algorithms have been described in [94] and [93], while an extension of these techniques based on MPC has been described in [56].
A distributed, partially connected and independent MPC algorithm for linear discrete-time systems has been described in [1], [2], where conditions for the a-posteriori stability analysis are given also in the case of communication failures among the local control units.
Finally, it is worth noting that the classification proposed here could be based also on other characteristics of the system under control, such as the presence/absence of coupled (joint) constraints in the various problem formulations.

[1] In game theory, a non cooperative Nash equilibrium of two (or more) players, is such that there no player can increase their utility pay-off by choosing a different strategy, given that any other player is going to change their strategy. A cooperative Pareto equilibrium is such that there is no other outcome that makes every player at least as well off and at least one player strictly better off. That is, a Pareto Optimal outcome cannot be improved upon without hurting at least one player.

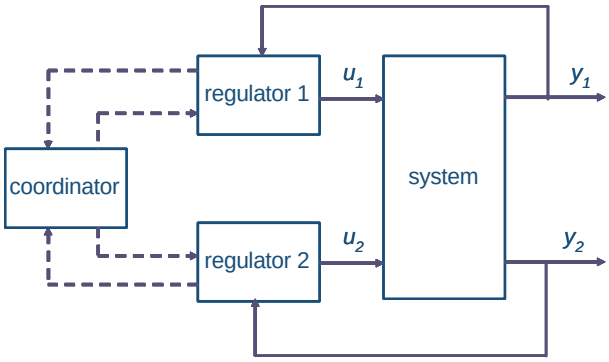

Figure 1.3: Hierarchical scheme for coordination of two local regulators.

1.3 Hierarchical control for coordination

An alternative to the distributed control schemes described in the previous section consists of considering a two level hierarchical control structure, such as the one shown in Figure 1.3, where an algorithm at the higher level coordinates the actions of local regulators placed at a lower level and possibly designed with MPC.

The design of the coordinator has been extensively studied over the last forty years, see e.g. the old but still fundamental books [57], [24]. The basic idea is to describe the overall system under control as composed by a number of subsystems linked through some interconnecting variables, i.e. the inputs of a given subsystem are the outputs of another one. Then, for any subsystem an optimization problem is solved with MPC by minimizing a suitable local cost function under local state, input and output constraints. If the computed local solutions satisfy the constraints imposed by the interconnecting variables, that is if there is coherence among the values of the interconnecting variables computed by the local regulators, the procedure is concluded. Otherwise, an iterative "price coordination" method is used: the coordinator sets the prices, which coincide with the Lagrange multipliers of the coherence constraints in the global optimization problem, by assuming as given the state, input and output variables defined by the local regulators. In turn, these optimal prices are sent to the low level local optimizers which take them as given and recompute the optimal trajectories of the state, input and output variables over the considered prediction horizon. The iterations are stopped when the interconnecting variables satisfy the required coherence conditions. This conceptual iterative procedure must be specialized to guarantee its convergence as well as some properties of the resulting final solution.
In the context of MPC, coordination schemes for discrete-time systems have been described in [62], [65], where also different communication schemes among the local regulators (agents) are considered. The proposed algorithms have been used for control of transportation networks, see [64], and power networks, see [63]. Another two layer structure developed with similar arguments has been presented in [38], which also describes an analogous two-level structure for the state estimation phase.
Finally, it must be noted that similar two-level structures are widely used in the intensive stream of research in computer science/artificial intelligence related to the so-called "autonomous agents". Basically, a number of agents (controllers) must negotiate their actions through a "negotiator" until a consensus on their actions is attained, see e.g. [3]. The ideas behind this approach have been specialized to the control design problem in [90].

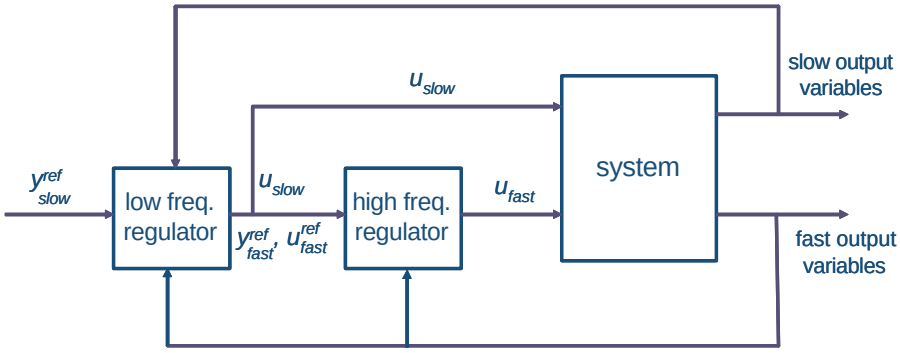

Figure 1.4: Control of a system with slow and fast dynamics.

1.4 Hierarchical control of multilayer systems

In hierarchical multilayer systems, the control action is performed by a number of regulators working at different time scales. This can be useful at least in two cases: when the overall process under control is characterized by different dynamic behavior, i.e. by slow and fast dynamics, or in plantwide optimization when optimization and control algorithms working at a different rates compute both the optimal targets and the effective control actions to be applied. These cases are considered in this section.

1.4.1 Hierarchical control of multi time scale systems

Many systems are characterized by clearly separable slow and fast dynamics, see e.g. [10] and [31] for a couple of significant industrial examples concerning a waste water treatment plant [10] and a greenhouse control problem [31]. In these cases, the control can be performed at two different time scales. A regulator acting at lower frequencies computes both the control actions of the manipulated variables which have a long-term effect on the plant, i.e. the "slow" control variables, and the reference values of the "fast" control variables, states and outputs. A second regulator takes these computed reference values as inputs and solves a tracking problem at a higher rate. A conceptual scheme of this architecture for a two layer structure is reported in Figure 1.4.
Two time scale systems are often referred to as singularly perturbed systems, and have been widely studied in the past, see e.g. [43]. However, in the context of MPC, systematic design methods guaranteeing well assessed properties are still lacking and only ad-hoc solutions tailored to some specific industrial problems have been described, see again [10] and [31]. In the development of new algorithms for these systems, one could take advantage from the multirate MPC methods developed in [82], [44], [85]. However, in these papers, the multirate nature of the problem usually stems from the adopted output sampling or input updating mechanisms.

1.4.2 Control of systems with hierarchical structure

Many industrial, economical or sociological systems can be described by a hierarchical structure, see e.g. the visionary book [57]. The highest layer of the hierarchy corresponds to a dynamical system with slow dynamics. This system can be controlled by looking at its behavior over a long time scale, and its computed control inputs must be effectively provided by subsystems placed at lower layers

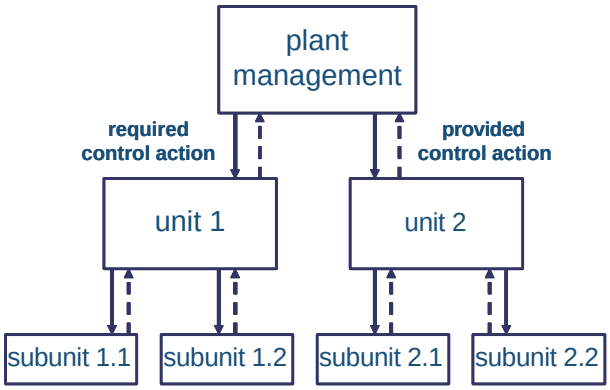

Figure 1.5: Hierarchical structure of a three layer system.

of the hierarchy and characterized by faster dynamics. In turn, these subsystems must be controlled at a higher rate and can be placed at an intermediate layer of hierarchy. An example of a three layer structure is reported in Figure 1.5.

As a matter of fact, in these structures the regulator at a higher layer computes its desired control inputs, which are the reference signals of the immediately lower layer. In order to guarantee that these computed references are feasible for the lower layer dynamics and constraints, as well as to consider the presence of disturbances acting at the lower layer, some additional information has often to be transmitted bottom-up. Moreover, the regulators of the subsystems at the lower layer must guarantee the solution of the corresponding tracking problems with an adequate level of accuracy, so that the mismatch between what is required by the higher level and what is provided by the lower one does not destroy some fundamental properties, such as stability and performance.
From a control engineering point of view, this multilayer hierarchical structure corresponds to a classical cascade feedback control system, see e.g. Figure 1.6 where again a three layer structure is considered and the inner loops correspond to faster dynamics, while the outer loop corresponds to the control of the system at the highest layer. In industrial control systems, the fastest dynamics is usually associated to the actuators, while the slowest one describes the process under control. The project of cascade control systems is typically made according to a frequency decoupling principle: the dynamics of the feedback loops are so different that in the design of the regulator for a given loop all the other loops can be assumed to be at the steady state. Moreover, no information is transmitted from inner to outer loops (dotted lines in Figure 1.6), so that any layer is unaware of the possibility of the lower layers to fulfill its requirements. In the design phase, the inner loops are often closed with standard PI-PID regulators, while MPC is used to design the control algorithm for the slowest system. When the frequency decoupling principle cannot be assumed, or when also the control of the subsystems at the lower layers of the hierarchy requires a more careful design, MPC can be used at any layer, with the clear advantage to consider the corresponding input, state and output constraints. Although this possibility has many potential advantages, few works have exploited it in depth. In particular, in [83] linear models are used to describe the systems at any layer and information is passed bottom-up to relax the requirements of the higher layer when infeasibility occurs at the lower layer. Overactuated cascade linear systems are analyzed in [84], while [70] deals with the tracking problem for plants at the higher layer described by Wiener models. In all these papers, the regulators at any layer are independently designed by resorting to robust MPC algorithms, so that the design phase turns out to

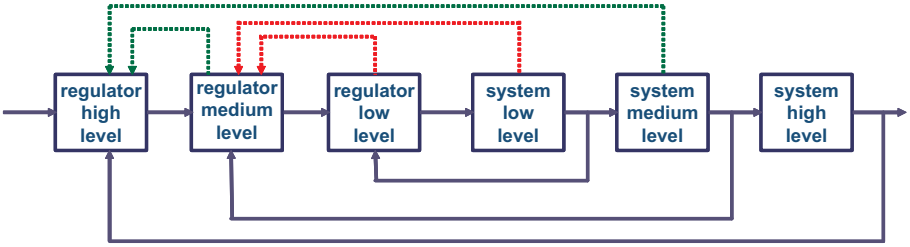

Figure 1.6: Three layer cascade control structure.

be completely decoupled even when the frequency decoupling principle does not hold.

1.4.3 Hierarchical control for plantwide optimization

In the process industry it is very common to design the overall control system according to the hierarchical structure shown in Figure 1.7, see e.g. [5], [86], [71]. At the higher layer, a Real Time Optimization (RTO) problem is solved to compute the optimal plant operating conditions with respect to a performance index usually representing an economic criterion. At this stage a detailed, although static, physical nonlinear model of the system is used. At the lower layer a simpler linear dynamic model of the same system, often derived by means of identification experiments, is used to design a regulator with MPC, guaranteeing that the target values transmitted from the higher layer are attained. Also in this case, the lower level can transmit bottom-up information on constraints and performance. Note that, although the previous approach is very popular in process control, in other contexts, such as in air traffic management systems, a somehow dual point of view is taken, see e.g. [69]. In these cases, at the higher level a simpler and more abstract model is considered to predict the long term behavior of the system and to optimize a given cost function over a long time horizon. At the lower level, a more accurate model is used to compute the current control actions by looking at a shorter time horizon. Also in this case, the lower level can transmit to the upper one information on constraints and performance.

In the multilayer structure of Figure 1.7, the design of the RTO module plays a fundamental role. In fact, even when it is based on a static model of the process, some main issues must be considered. First, the adopted model has to be periodically updated (adapted) by means of some kind of reconciliation procedure to deal with changing operating conditions due to slow disturbances. Second, coherence must be guaranteed between the more abstract model (typically a low order one) used in the design phase at the upper layer and the model (a more complex one) used at the lower layer for the MPC implementation, see e.g. [98]. Third, accurate steady-state target optimization must be done to guarantee that the input and output steady state references computed by RTO are feasible and as close as possible to the desired set-points, see e.g. [75] and the results recently reported in [77], [76] to solve this feasibility problem.

Many papers have been published in the MPC literature dealing with the hierarchical structure of Figure 1.7. Among them, a recent and interesting survey on the subject is reported in [91], where also a wide list of references is provided. It is also worth recalling [20], where a thorough discussion on the merits, limitation and implementation aspect of RTO is reported. A RTO procedure based on a dynamic model of the process is described in [36]. An attempt to mix the two layers of the hierarchy

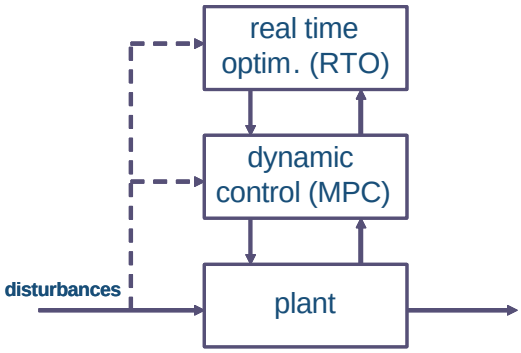

Figure 1.7: Hierarchical structure for plantwide control and optimization.

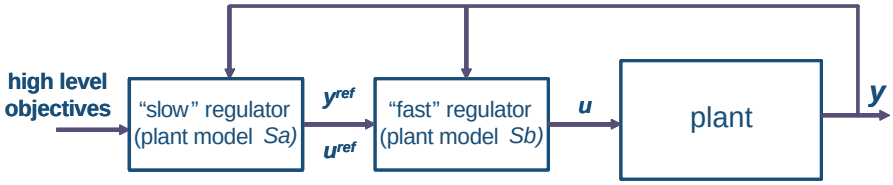

Figure 1.8: Hierarchical control structure for plantwide control and optimization.

of Figure 1.7, i.e. to integrate (nonlinear) steady-state optimization and (linear) MPC control is described in [99]. Steady-state target calculation for a set of local MPC regulators has been considered in [12] by adopting an approach based on coordination and similar to the one discussed in Section 4, while a high level coordinator maximizing the plant throughput based on the information provided by lower level local MPC regulators has been described in [4].
Despite the large amount of results on RTO, it is believed that much work has still to be done to extend many theoretical results (stability, performance, robustness) nowadays available for standard MPC implementations to the considered hierarchical structure, see e.g. [54], [53].

Finally, a couple of remarks are in order. First, it can be noted that the conceptual scheme of Figure 1.7 can be given the equivalent and more "control oriented" representation of Figure 1.8, where a two layer structure is considered and each layer uses a different system model (*Sa* and *Sb*) in the design of the corresponding regulator. Second, it is worth pointing out that the conceptual classification adopted here, which distinguishes between the schemes of Figures 1.5 and 1.7, is not always clear in the technical literature. In fact, a very popular picture is the one depicted in Figure 1.9 where the regulators (PI-PID) at the lowest layer control the actuators, so that they make reference to the actuators' models (as in Subsection 1.4.1), while conceptually, the two higher levels make reference to the plantwide optimization problem described in this subsection.

1.5 Coordinated control of independent systems

So far, large scale systems made by interacting subsystems have been considered. Another significant scenario is related to the problem of coordinating a number of decoupled systems (agents) which must cooperate to achieve a given goal, i.e. to globally minimize a cost function subject to joint constraints.

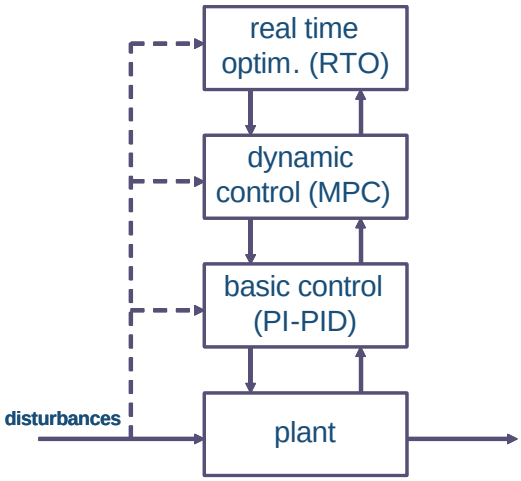

Figure 1.9: A popular representation of plantwide control.

Also in these cases, instead of solving a unique centralized control problem, in principle it is possible to solve a number of local optimization problems and to coordinate the local actions of the agents by means of a suitable exchange of information. It is apparent that the design of local, but coordinated control algorithms shares many features with the problem considered in Section 1.2, so that the more detailed classifications proposed there will not be replicated here.
In the context of MPC, the coordination of independent nonlinear discrete time systems with delayed intercommunication has been considered in [25], which also provides stability results obtained by resorting to Input to State Stability (ISS) concepts. In [40], [39], any agent (a node of a graph) is described as a discrete-time nonlinear system which knows the state of its neighborhoods without delay. The local performance indices weight the state and the inputs of the neighborhoods and the local future control values are computed over the considered horizon to predict the transient of the local state. Stability is achieved with a zero terminal constraint, as usual in MPC, see [54]. The problem considered in [79] consists of controlling a number of disturbed subsystems described by discrete-time linear models with independent dynamics but with coupling constraints. The proposed solution is based on a non iterative procedure where any regulator solves its own optimization problem (with local performance indices) but knows the most recent or the predicted plans for the other subsystems. Constraint satisfaction is guaranteed by a constraint tightening procedure similar to the one already used in [13] and in [46]; feasibility and convergence are guaranteed. An extension of this approach based on recent results on robust MPC design (see [55]) is reported in [92]. The formation control of vehicles with independent second order nonlinear continuous-time dynamics and coupling cost functions has been solved with the Receding Horizon approach also in [19], while in [27] a similar problem (independent cost functions) is solved for double integrators by looking to a Nash equilibrium solution.
In all the above mentioned approaches the final goal, for example the equilibrium point to be reached, is specified a-priori. On the contrary, in another class of coordinated control problems the subsystems must negotiate on-line their final outcome starting from a partial knowledge of the overall system, for example the state of their neighbors. These are usually called “consensus problems” and have a wide number of potential applications, such as flocking, rendezvous, formation control and alignment problems or coordination of sensor networks. Among the ever increasing number of publications on

this subject, it is possible to recall here that the problem has been formulated and solved in [32] for systems described by a single integrator, while fundamental contributions have been given in [59]. In the context of MPC, preliminary results have been reported in [35], while the consensus problem for single and double integrator dynamics has been solved with stabilizing MPC approaches in [21], [22], [23].

1.6 Related design problems and conclusions

This chapter has reviewed a number of architectures and algorithms for control of large scale systems. Concerning the implementation of the proposed distributed and hierarchical solutions, many fundamental problems must be considered to obtain an adequate level of performance. Among them, the following are of paramount importance.

- *New algorithms with guaranteed properties*. Many theoretical contributions are required to develop efficient algorithms with guaranteed properties, such as stability and performance. In particular, this is true for decentralized MPC, where very few results are available, and for the design of hierarchical MPC regulators for multilayer systems. It is believed that in all the considered cases, one could take advantage of recent results on robust MPC, see e.g. [53], and on the analysis of interconnected systems with a "small gain" approach, see [15]. However, robust MPC and small gain properties naturally lead to very conservative results, with performance not acceptable in real world applications.

- *Selection of the control structure*. Criteria must be developed for the selection of the proper control structure based on the relative improvements achievable by increasing the complexity. For example, it is apparent that a distributed controller can stabilize systems which cannot be stabilized by a decentralized one because of the presence of fixed modes. However, if stability can be provided by both the schemes, it is still to be evaluated in term of performance whether it is worth considering a more complex structure, which requires more information to be transmitted among local control units. A second issue concerns the comparison of the performance provided by distributed regulators (see Section 1.2) with respect to those achievable with the hierarchical approach for coordination described in Section 1.3.

- *Reconfigurable control structures and hybrid systems.* With reference to the hierarchical structures described in Section 1.4.2, one should explore the possibility to reconfigure the system, for example by adding or removing actuators and sensors ("plug and play control", see [42]). This could be useful to consider time varying performance requirements and to control systems described by a hybrid model. For example, consider the problem of the optimal management of the start-up of a thermal power plant. During this phase, the control configuration and the control objectives are usually very different from those to be considered during standard operating conditions. Preliminary work on hierarchical control of hybrid systems has been described in [49], where however non predictive approaches have been taken. Finally, a flexible control configuration can better cope with the requirement of a high tolerance to faults.

- *Optimization algorithms*. Many optimization algorithms have been developed to solve efficiently the minimization problems related to linear and nonlinear centralized MPC, see e.g. [9], [16]. On the contrary, optimization methods for distributed and hierarchical MPC are still lacking. This is an important and critical point where significant improvements are expected.

- *Distributed state estimation*. Distributed control algorithms call for the availability of distributed state estimators guaranteeing the asymptotic convergence of the local state estimates. Preliminary results have been reported in e.g. [68], [67], [37], where sensor networks have been considered. However, further developments are required to include in the state estimation problem the knowledge on state and noise constraints, as well as to link the convergence properties of the local estimates to local or global observability properties. A predictive approach can be taken also for this problem by resorting to the ideas underlying the moving horizon estimators described in [58], [73], [74].

- *System partitioning*. In the design of decentralized and distributed control systems (Sections 1.1 and 1.2), eventually coordinated as described in Section 1.3, the process under control must be a-priori partitioned into subsystems properly defined to reduce the dynamic couplings and to facilitate the control design. In some cases partitioning is natural in view of the process layout, for example chemical plants are often composed by a series of process units with some recirculating products, see [78]. In other cases, the partitioning can be made by means of an input-output analysis based on the Relative Gain Array and related indices, see [89], [26], [66], [28], [28], [30], or on an analysis in the state space based on gramians, see e.g. [97], [80]. Temporal decomposition and model reduction, useful for the design of the hierarchical control systems described in Section 1.4, can be performed with singular value decomposition, see again [89].

- *Synchronization and communication protocols.* Whenever the adopted control structure requires an exchange of information among local regulators, at the same or at different layers of a hierarchical structure, the achievable performance strongly depends on the adopted implementation (see e.g. the discussion in [65]) and communication protocols. Moreover, some fundamental problems related to these aspects must be considered, such as low transmission frequency or loss of information. The interested reader is referred to [6] for an insightful discussion on these aspects.

Chapter 2

Partitioning methods for distributed and hierarchical control

This chapter surveys a number of approaches for the solution of the following problems for linear systems:

- decomposition of a dynamical system into a number of weakly interacting subsystems;
- representation of a dynamical system at different levels of abstraction for the design of decentralized/distributed or hierarchical control systems.

These problems can be solved according to different rules and goals:

1. A functional/spatial decomposition aimed at minimizing the control system complexity while still guaranteeing a given level of performance. In this context, it is necessary to choose the proper controlled outputs; to select the inputs to manipulate; to partition the system into weakly interacting subsystems, to define the control structure, to synthesize the control law.
2. A temporal decomposition where different dynamic behaviors (fast/slow) of the system must be recognized so as to facilitate the synthesis of controllers working at different time scales. Another important problem concerns the representation of the system at different levels of abstraction where the higher levels describe the slow system dynamics. This representation naturally leads to the design of hierarchical control systems where the top control level defines the system operating conditions usually according to economic criteria, while the lower levels are more related to the control of the plant units.

The chapter is organized as follows. In Section 2.1 the methods based on the Relative Gain Array (RGA) for the design of decentralized controllers are described. Section 2.2 presents the decomposition approaches based on the analysis of the controllability and observability gramians. Finally, Section 2.3 is devoted to describe the most popular and effective model reduction techniques.

2.1 Decomposition methods based on the Relative Gain Array

Consider a linear, continuous-time, invariant and square system described by

$$\Sigma : \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \tag{2.1}$$

where $x(t) \in \Re^n$ is the state, $u(t) \in \Re^m$ is the input variable and $y(t) \in \Re^m$ is the output. The transfer function of the system is $G(s) = C(sI - A)^{-1}B + D$, with steady-state gain $G_0 = G(0)$, whose individual elements are denoted by g_{ij}, $i, j = 1, 2, \ldots, m$.

Assumption 1 *$G(s)$ has stable poles and no invariant zeros at the origin.*

2.1.1 Relative Gain Array

The system Relative Gain Array (RGA), in the following denoted by Λ, is defined as

$$\Lambda = G_0 \times (G_0^{-1})' \tag{2.2}$$

where the symbol $\times$ represents the element by element multiplication. The elements

$$\lambda_{ij} = \frac{(\partial y_i / \partial u_j)_{u_{l \neq j} constant}}{(\partial y_i / \partial u_j)_{y_{k \neq i} constant}} \tag{2.3}$$

of the matrix Λ represent the ratio between the process gain for the pairing $y_i - u_j$ in an isolated loop and the process gain in the same loop when all other loops are closed. As discussed in [89], the matrix Λ has a number of interesting properties. Among them:

- its elements are independent of the adopted units;
- the sum of the elements of any row is equal to 1;
- the sum of the elements of any column is equal to 1;
- it is equal to the identity if $G(s)$ is a diagonal or a triangular matrix.

In the selection of the pairings among input and output variables for the design of SISO (Single Input Single Output) decentralized controllers, it is advisable to select those pairs that maintain roughly the same gain in open-loop and closed-loop configurations, i.e. $\lambda_{ij} \simeq 1$, while the pairings for which a change of sign of these gains occurs, i.e. $\lambda_{ij} < 0$, must be avoided. By recalling the meaning of the elements of Λ, the two following main rules are recommended:

i) select the input-output pairs such that the corresponding elements of Λ are positive and close to 1;

ii) avoid the input-output pairs corresponding to negative elements of Λ.

Extensions of the basic definition to consider

- nonsquare systems,
- analysis of the coupling at a given frequency,

are also reported in [89], together with the analysis of the connections between the Relative Gain Array and the achievable performances of decentralized control schemes. Concerning this point, in the wide literature available, it is worth mentioning [26], where a wide list of references is also given.

2.1.2 Niederlinski index

In many cases, from the analysis of the RGA it is not possible to find a unique, dominating, solution to the problem of pairing input and output variables, but a number of apparently equivalent solutions can be recognized. In these cases, it is convenient to use the Niederlinski index (NI), see [66], defined as follows. For a given choice of the input-output pairings, let $\overline{G}_0$ be the matrix obtained from G_0 by setting to zero all the elements which do not correspond to the selected m input-output pairs. The Niederlinski index $N_{\overline{G}_0}(G_0)$ is

$$N_{\overline{G}_0}(G_0) = \frac{\det(G_0)}{\det(\overline{G}_0)} \tag{2.4}$$

and the following criterion can be followed:
among the possible sets of pairings selected by looking at the RGA, choose the pairings with a positive Niederlinski index.
It can be proved that a decentralized control configuration corresponding to a positive value of $N_{\overline{G}_0}(G_0)$ has the potentiality to be *Integral Controllable with Integrity* (ICI), where the ICI property is defined as follows:

Definition 1 *The system is* Integral Controllable with Integrity *(ICI) if there exists a controller such that the closed loop system is stable and such that each individual loop may be detuned independently by a factor ε_i, $\varepsilon_i \in [0,1]$, without introducing instability.*

2.1.3 Partial Relative Gain

The method has been proposed in [28] to provide necessary conditions for a control configuration to be ICI and to derive pairing rules in cases where the conventional use of the RGA fails or is ambiguous. Moreover, it can be useful to select cases where block-decentralized control structures must be preferred to those based on conventional SISO regulators. In order to introduce the Partial Relative Gain (PRG) index, first assume that the matrices G_0 and Λ are partitioned as follows

$$G_0 = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}, \Lambda = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{bmatrix} \tag{2.5}$$

where G_{11} and Λ_{11} have the same dimensions, while G_{22} is assumed to be nonsingular. It is possible to show that

$$\Lambda_{11} = G_{11} \times (\overline{G}_{11}^{-1})', \overline{G}_{11} = G_{11} - G_{12}G_{22}^{-1}G_{21} \tag{2.6}$$

The matrix $\overline{G}_{11}$ is the gain matrix of the subsystem $G_{11}(s)$ when the rest of the system, i.e. $G_{22}(s)$, is closed under integral feedback control. Then, the procedure based on the PRG approach can be summarized as follows:

- from the analysis of the RGA choose an input-output pairing (i.e. $u_j - y_i$),
- reorder the matrix G_0 as in (2.5) so that G_{22} corresponds to the selected input-output pair,
- recompute the RGA for the other subsystem (G_{11}) given that G_{22} is in an integral feedback loop, i.e. $\Lambda_P = \overline{G}_{11} \times (\overline{G}_{11}^{-1})'$,
- from the analysis of Λ_P choose a new pairing,

- repeat the procedure until all the $u-y$ pairs have been chosen.

In so doing, the selection of the input-output pairs is done by considering that some loops have been already closed, that is PRG is an attempt to consider closed-loop information related to the decentralized control structure progressively chosen. Note however that the procedure is not uniquely defined, since the (initial) choice of the pair $u_j - y_i$ heavily influences all the subsequent steps. In any case, the procedure closely resembles a typical approach in the design of a decentralized closed-loop control system: once a loop has been closed, the project of the regulators for the other loops considers the closed-loop system already defined. Finally, note that necessary conditions for ICI in terms of PRG have been reported in [28].

2.1.4 Decomposed Relative Interaction Array

The Decomposed Relative Interaction Array (DRIA) is a method, first proposed in [29], aimed at refining the results of the analysis performed with the RGA and NI indices. Associated to any element of the static gain matrix G_0 it is possible to define the matrix

$$\Psi^{ij} = \Delta G_{ij} \times (G_{ij}^{-1})' \tag{2.7}$$

where G_{ij} is the matrix of the static gains of the overall system with the i-th row and the j-th column removed, while ΔG_{ij} is the incremental process gain matrix of subsystem G_{ij} when loop $y_i - u_j$ is closed, that is

$$\Delta G_{ij} = -\frac{1}{g_{ij}} g_{\bullet j}^{ij} g_{i\bullet}^{ij}$$

where g_{ij} is again the (i,j) element of G_0 and the vectors $g_{\bullet j}^{ij} g_{i\bullet}^{ij}$ are the ith row and jth column of G_0 with the element g_{ij} removed. Then, $m \times m$ DRIA matrices are associated to the system, each one of them with dimension $m-1$. Recalling that for any matrix Υ the 2-norm is defined by

$$\|\Upsilon\|_2 = \rho(\Upsilon)^{1/2}$$

where $\rho(\Upsilon)$ is the spectral radius of Υ, it is possible to compute a new matrix Ω whose element (*i*,*j*) is

$$\omega_{ij} = \|\Psi^{ij}\|_2$$

Finally, in addition to the previous rules concerning the analysis of RGA and NI, in [29] it is suggested to consider the following one:

- choose the pairings that correspond to the smallest ω_{ij} values

2.1.5 Decomposed Relative Gain Array

The Decomposed Relative Gain Array (DRGA), proposed in [30], is a technique to evaluate the effectiveness of a loop pairing previously selected with the criteria based on RGA, NI, DRIA. The procedure to compute the DRGA matrix Γ can be summarized as follows:

- select by means of RGA, NI and DRIA a given pairing of input and output variables,
- rearrange the matrix $G(s)$ into $G_r(s)$ so that the selected pairings correspond to the diagonal elements of $G_r(s)$,

- compute the Relative Gain Array matrix Λ_r of $G_r(s)$,
- compute the DRGA matrix as $\Gamma = 0.5(diag(\Lambda_r)^{-1}\Lambda_r + \Lambda_r diag(\Lambda_r)^{-1})$.

The matrix Γ has the following properties:

i) it depends only on the steady-state gain of the system;

ii) it is scaling independent;

iv) if $G_r(s)$ is diagonal or triangular, Γ is a zero matrix.

It is now possible to define the *Relative Control Performance Index* (RCPI), i.e. a tuning knob $\varepsilon \in [0,1]$, which indicates the confidence level on system overall control structures with respect to the promising control structures.

The value of ε can be chosen according to the following design rules:

- $\varepsilon = 0$: a centralized control structure is preferred regardless of the complexity of the process;
- $\varepsilon = 1$: a fully decentralized controller is preferred;
- for small values of ε a block diagonal controller or a sparse controller are preferred.

Once the value of ε has been chosen, denoting by γ_{ik} the (*i*,*k*) element of Γ and by g^r_{ik} the (*i*,*k*) element of $G_r(s)$, a dominant model $\widetilde{G}_r(s)$ to be used in the control design can be selected as follows:

$$\widetilde{G}_r(s) = \{g^r_{ik} | g^r_{ik} = 0, if |\gamma_{ik}| < \varepsilon, i,k = 1,2,\ldots,n\} \tag{2.8}$$

Therefore, the value of the parameter ε influences the choice of the single-input, single-output transfer functions which are neglected in the definition of the dominant model The structure of the resulting dominant model $\widetilde{G}_r(s)$. In turn, this choice obviously strongly impacts on the following control design phase.

2.2 Decomposition methods based on Gramians

In order to quantify the interaction between a control loop and the others it is possible to use the notions of gramians, which describe controllability and observability properties of a given stable linear system, see [97], [80].

Definition 2 *For a stable system (2.1), the controllability gramian P and the observability gramian Q are symmetric non negative definite matrices which satisfy the Lyapunov equations*

$$AP + PA' + BB' = 0 \qquad A'Q + QA' + C'C = 0$$

Alternatively, the matrices P and Q can be expressed as

$$P = \int_0^\infty e^{At} BB' e^{A't} dt \qquad Q = \int_0^\infty e^{A't} C'C e^{At} dt$$

The gramians quantify how hard it is to control and to observe the system states, and the ranks of P and Q are related to the dimensions of the controllable and observable subspace respectively.
Now consider the product PQ, it can be proven that its eigenvalues λ_i, $i = 1,2,\ldots,n$, are non negative and do not depend on the particular realization.

Definition 3 *The* Hankel Singular Values *(HSV) of system (2.1) are defined as*

$$\sigma_H^{(i)} = \sqrt{\lambda}_i \quad i = 1,\ldots,n \tag{2.9}$$

where the λ_i *are ordered to obtain* $\sigma_H^{(1)} \geq \sigma_H^{(2)} \geq \ldots \geq \sigma_H^{(n)}$.

Note that the HSV can be related to a system norm through the following definition.

Definition 4 *The* Hankel Norm *of a system with transfer function* $G(s)$ *is defined as*

$$\|G(s)\|_H = \sqrt{\lambda_{\max}(PQ)} = \sigma_H^{(1)} \tag{2.10}$$

Let b_j be the $j-th$ column of matrix B and c_i the $i-th$ row of C. Define by P_j and Q_i the controllability and observability gramians for the elementary system $(A, b_j, c_i, 0)$. Then the original system controllability and observability gramians P and Q can be written as (see [80])

$$P = \Sigma_{j=1}^{m} P_j \quad Q = \Sigma_{i=1}^{m} Q_i$$

and

$$PQ = \Sigma_{j=1}^{m} \Sigma_{i=1}^{m} P_j Q_i$$

2.2.1 Participation Matrix

As shown in [80], in order to quantify the interaction between the $j-th$ input and the $i-th$ output it is possible to consider the trace of the matrix $P_j Q_i$, which has the following properties:

i) the trace of any $P_j Q_i$ is non negative and state realization independent;

ii) the trace of any sum of terms $P_j Q_i$ is the sum of the traces of the individual terms;

iii) the trace of any sum of terms $P_j Q_i$ is monotonically non decreasing when new terms are added;

iv) the trace of $P_j Q_i$ is equal to the sum of the squared HSV for the system with transfer function $G(s)_{ij}$

v) the trace of PQ is larger than, or at least equal to, the trace of any sum of terms $P_j Q_i$.

Define now the *Participation Matrix* (PM) whose elements are

$$\Phi = \{\phi_{ij} | \phi_{ij} = \frac{trace(P_j Q_i)}{trace(PQ)}\} \quad 0 < \phi_{ij} < 1 \tag{2.11}$$

with the property

$$\Sigma_{i=1}^{m} \Sigma_{j=1}^{m} \phi_{ij} = 1$$

It follows that the "average" value of any element ϕ_{ij} of PM is $1/m^2$, while if the element (*i*,*j*) of PM is high (near to 1), the *j-th* input has a great influence on the *i-th* output. On the contrary, if

$\phi_{ij} \simeq 0$ the effect of u_j over y_i is minor. In view of this consideration, an approximate simplified model $G_r(s)$ of the system can be obtained by forcing the elements $g_{rij}(s)$ of $G_r(s)$ to be equal to $g_{ij}(s)$ if $\phi_{ij} \geq \varepsilon$, $1 > \varepsilon \geq 0$ being a tuning knob which can be chosen equal to $1/m^2$, and by setting them to zero otherwise. Denoting by Σ the sum of the elements ϕ_{ij} corresponding to the input-output pairs included in the model $G_r(s)$, an empirical rule of thumb to evaluate the quality of the model is to verify that Σ is greater than a given value, typically 0.7, while for $\Sigma < 0.5$ the quality of the approximation is usually unacceptable.

Obviously, once the model $G_r(s)$ has been chosen, its zero/nonzero pattern forces the structure of the regulator to be designed.

For discrete time systems described by

$$\Sigma : \begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) + Du(k) \end{cases} \tag{2.12}$$

all the previous considerations hold as well, but the gramians must be computed from the equations

$$A'PA - P + BB' = 0 \qquad AQA' - Q + C'C = 0$$

Moreover, if the system has input or output time delays, some further considerations can be done to refine the analysis, see again [80].

2.2.2 Hankel Interaction Index Array

Strictly related to the PM, it is possible to consider the Hankel Norm of the transfer function $G(s)$ of the system, as shown in [97]. To this end, consider the *Hankel interaction index array* $\bar{\Sigma}_H$ whose elements are

$$\bar{\Sigma}_{Hij} = \|G_{ij}(s)\|_H$$

where $G_{ij}(s)$ is the $(i,j) - th$ element of $G(s)$.

Matrix $\bar{\Sigma}_H$ can be interpreted as a gain matrix relating the Hankel norms of inputs and outputs. Similarly to the participation matrix PM, it takes the full dynamic effects of the system into account and does not only focus on the steady-state performance as the RGA does.

To eliminate the effect of scaling, one can normalize the matrix in different ways, for example by considering (as for RGA) the matrix

$$\bar{\Sigma}_H \times (\bar{\Sigma}'_H)^{-1}$$

whose rows (columns) have elements which sum to one. From the analysis of the Hankel interaction index array it is finally possible to choose the most appropriate control structure (fully decentralized, sparse, centralized).

2.3 Model order reduction

The project of hierarchical systems where the higher levels of the hierarchy are designed on the basis of simplified models of the plant under control calls for model reduction techniques. In this section, the main algorithms for model reduction are briefly summarized. The presentation strictly follows the

results reported in [89]. Let (2.1) be a balanced realization of the system under control, so that its controllability and observability gramians are such that

$$P = Q = \Sigma = diag(\sigma_1, \sigma_2, \ldots, \sigma_n),\ \sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_n > 0 \tag{2.13}$$

Moreover, partition the system matrices as

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, C = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \tag{2.14}$$

where $A_{11} \in R^{k,k}$, $B_1 \in R^{k,m}$, $C_1 \in R^{m,k}$. Correspondingly, the matrix Σ is

$$\Sigma = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \tag{2.15}$$

where $\Sigma_1 \in R^{k,k}$.

The reduced order model obtained by *balance truncation* is then described by the matrices (A_{11}, B_1, C_1, D). It is a balanced realization itself and its singular values are those of Σ_1. Moreover, letting $G_a^k(s)$ the corresponding transfer function, it results that

$$\|G(s) - G_a^k(s)\|_\infty \leq 2(\sigma_{k+1} + \sigma_{k+1} + \ldots + \sigma_{k+n}) \tag{2.16}$$

Another possibility for model reduction consists of considering the balanced residualization technique, where it is assumed $\dot{x}_2 = 0$ (the "fast" dynamics is supposed to be always at the steady state, as it is done in the analysis of singularly perturbed systems). Under this hypothesis, the reduced order model is

$$\Sigma : \begin{cases} \dot{x}_1(t) = A_r x_1(t) + B_r u(t) \\ y(t) = C_r x_1(t) + D_r u(t) \end{cases} \tag{2.17}$$

where

$$A_r = A_{11} - A_{12}A_{22}^{-1}A_{21},\ B_r = B_1 - A_{12}A_{22}^{-1}B_2 \tag{2.18}$$

$$C_r = C_1 - C_2A_{22}^{-1}A_{21},\ D_r = D - C_2A_{22}^{-1}B_2 \tag{2.19}$$

Balanced residualization enjoys the same error bound (2.16) of balance truncation. Moreover the corresponding transfer function $G_a^k(s)$ has the same static gain of the original system, i.e. $G_a^k(0) = G(0)$, so that it must be preferred to balanced truncation when an accurate model at low frequency is required.

Chapter 3

A chemical benchmark for distributed and hierarchical control

This chapter describes the dynamic model of a large chemical plant typical of many industrial applications. The plant is composed by three binary distillation columns, three chemical reactors and six chemical components. This system has been first studied in [48] and represents an interesting benchmark for plantwide control of complex processes.

3.1 Dynamic model of the plant units

3.1.1 Dynamic model of the reactors

Consider a chemical reactor and assume that:

- all the energy phenomena are negligible;
- the hydraulic phenomena are all at the steady state;
- perfect mixing inside the reactor;

Define:

- q_{I_j} volumetric flow rate of the $j-th$ input;
- $c_{I\,ji}$ concentration of the $i-th$ component in the $j-th$ input flow rate;
- V reactor volume;
- c_i concentration inside the reactor of the $i-th$ component;
- q_{O_j} volumetric flow rate of the $j-th$ output;
- $c_{O\,ji}$ concentration of the $i-th$ component in the $j-th$ output flow rate;
- n_i number of input components;
- n_0 number of output components;
- n_r number of reacting components;

- k reaction constant;

The mass balance of the $i-th$ component inside the reactor is then given by

$$\frac{dc_i(t)}{dt} = \frac{1}{V} \cdot \left[\sum_{j=1}^{n_i} c_{I\,ji} \cdot q_{I\,j} - \sum_{j=1}^{n_o} c_{O\,ji} \cdot q_{O\,j} \right] \pm k \cdot \prod_{r=1}^{n_r} c_r \tag{3.1}$$

Assuming that inside the reactor there are n components, the model will be described by a system of n differential equations besides one more equation describing the hydraulic equilibrium, that is

$$\sum_{j=1}^{n_i} q_{I\,j}(t) = \sum_{j=1}^{n_o} q_{O\,j}(t) \tag{3.2}$$

Finally, note that the dynamic model previously derived can be expressed in terms of molar fractions x_i, instead of concentrations c_i by defining

$$x_i = \frac{c_i}{\sum_{j=1}^{n} c_j} \tag{3.3}$$

Then, with an obvious meaning of symbols, the dynamic equations can be written as

$$\frac{dx_i(t)}{dt} = \frac{1}{V} \cdot \left[\sum_{j=1}^{n_i} x_{I\,ji} \cdot q_{I\,j} - \sum_{j=1}^{n_o} x_{O\,ji} \cdot q_{O\,j} \right] \pm k \cdot \prod_{r=1}^{n_r} x_r \tag{3.4}$$

and

$$\sum_{j=1}^{n} x_j = 1$$

3.1.2 Dynamic model of the columns

The simplified model of the tray distillation column here considered assumes that it is composed by five sections:

1. condenser
2. enriching section
3. feed tray
4. stripping section
5. reboiler

where the enriching and stripping sections can be composed by a variable number of trays. A schematic diagram of the column is shown in Figure 3.1, where:

1. V is the vapor flow rate;
2. R is the reflux flow rate;
3. D is the distillate flow rate;

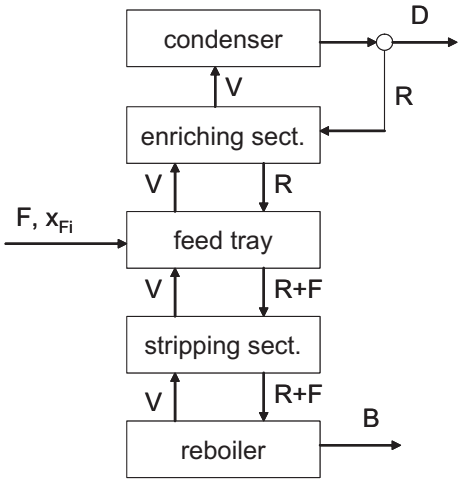

Figure 3.1: Schematic representation of a distillation column

4. B is the flow rate of the bottom product;
5. F is the feed flow rate;
6. x_{fi} is the liquid molar fraction of the $i-th$ component into the feed flow rate.

Assume that the mixture is formed by N components and let

- x_i liquid molar fraction of the $i-th$ component ($i = 1,2,\ldots,N$);
- y_i vapor molar fraction of the $i-th$ component ($i = 1,2,\ldots,N$);
- α_i volatility of the $i-th$ component ($i = 1,2,\ldots,N$);
- α_{ij} relative volatility of the $i-th$ component with respect to the $j-th$ component ($i,j = 1,2,\ldots,N$).

Straightforward computations allow to conclude that the relation among the liquid and the vapor molar fractions is given by the following set of linear equations

$$\begin{pmatrix} 1+\alpha_{1N}\frac{x_1}{x_N} & \alpha_{1N}\frac{x_1}{x_N} & \cdots & \alpha_{1N}\frac{x_1}{x_N} \\ \alpha_{2N}\frac{x_2}{x_N} & 1+\alpha_{2N}\frac{x_2}{x_N} & \cdots & \alpha_{2N}\frac{x_2}{x_N} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{(N-1)N}\frac{x_{N-1}}{x_N} & \cdots & \cdots & 1+\alpha_{(N-1)N}\frac{x_{N-1}}{x_N} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix} = \begin{pmatrix} \alpha_{1N}\frac{x_1}{x_N} \\ \alpha_{2N}\frac{x_2}{x_N} \\ \vdots \\ \alpha_{(N-1)N}\frac{x_{N-1}}{x_N} \end{pmatrix} \tag{3.5}$$

The mathematical model of the column is derived under the fundamental assumption that the energetic phenomena are negligible, so that only mass balance equations are used. Moreover, the following simplifying hypothesis are introduced.

- the pressure inside the column is constant;
- the vapor flow rate V can be directly manipulated (the reboiler has no dynamics);

- the liquid (R) and vapor (V) flow rates are constant inside the column;
- the hydraulic dynamics is negligible with respect to the dynamics of the concentrations;
- the vapor hold-up on the trays is negligible with respect to the liquid hold-up;
- the Murphee efficiency is constant for any ($i-th$) component and any ($j-th$) tray, that is

$$E_{ji} = \frac{y_{ji} - y_{(j-1)i}}{y^*_{ji} - y_{(j-1)i}} = 1 \tag{3.6}$$

where y^*_{ji} is given by:

$$y^*_{ji} = \frac{\alpha_{iN} x_i}{1 + (\alpha_{iN} - 1) x_i} \tag{3.7}$$

Define the following quantities:

- H_j liquid hold-up in the $j-th$ tray;
- H_a liquid hold-up in the feed tray;
- N_a number of trays in the enriching section;
- N_e number of trays in the stripping section;
- x_{ji} liquid molar fraction of the $i-th$ component in the $j-th$ tray;
- y_{ji} vapor molar fraction of the $i-th$ component in the $j-th$ tray;
- x_{ai} liquid molar fraction of the $i-th$ component in the feed tray;;
- x_{ti} liquid molar fraction of the $i-th$ component in the top product;
- x_{bi} liquid molar fraction of the $i-th$ component in the bottom product;
- F feed flow rate;
- x_{fi} liquid molar fraction of the $i-th$ component in the feed flow rate;
- α_{iN} relative volatility of the $i-th$ component with respect to the $N-th$ component.

Denoting by the index $j = 1$ the reboiler and by the index $j = N_p$ the condenser and defining:

- $x_{1i} = x_{bi}$
- $x_{Np_i} = x_{ti}$
- $N_p = N_a + N_e + 3$ total number of trays, including reboiler and condenser

the mass balance for any tray and for any $i-th$ component is:

1. Static balance of the flow rates at the condenser:

$$V = R + D$$

2. Static balance of the flow rates at the reboiler:

$$V + B = R + F$$

3. Dynamic balance at the reboiler:

$$H_{1i}\dot{x_{1i}} = -Vy_{1_i} + (R+F)x_{2i} - Bx_{1i}$$

4. dynamic balance in the stripping section (j=[2, N_e+1]):

$$H_{j_i}\dot{x_{j_i}} = (R+F)(x_{(j+1)_i} - x_{j_i}) + V(y_{(j-1)_i} - y_{j_i})$$

5. dynamic balance at the feed tray:

$$H_a\dot{x_{ai}} = Rx_{(N_e+3)_i} - (R+F)x_{ai} + V(y_{(N_e+1)_i} - y_{a_i}) + Fx_{f_i}$$

6. dynamic balance in the enriching section (j=[N_e+3, N_p-1]):

$$H_j\dot{x_{j_i}} = R(x_{(j+1)_i} - x_{j_i}) + V(y_{(j-1)_i} - y_{j_i})$$

7. dynamic balance at the condenser:

$$H_{Np}\dot{x_{Np_i}} = Vy_{(Np-1)_i} - (R+D)x_{Np_i}$$

The complete model of a distillation column with N_p trays is then described by the above seven equations, written for any component, besides the additional relations (3.5), (3.6) and (3.7).

3.2 Plant description

The plant is composed by three reactors, $R1$, $R2$ and $R3$, three distillation columns, $C1$, $C2$ and $C3$, two recycle streams and six chemical components, named A, B, C, D, E, F. The flow diagram of the plant is shown in Figure 3.2.

3.2.1 Plant data and nominal inputs

The main plant variables used to obtain the results discussed in the following are now summarized. These values can be easily modified by editing the Matlab files described in the Appendix.

Relative volatilities

$$\alpha_{AF} = 8,\ \alpha_{BF} = 6,\ \alpha_{CF} = 4,\ \alpha_{DF} = 2,\ \alpha_{EF} = 1.2$$

Reactor R1

$k_1 = 10[h^{-1}]$ reaction constant;

$V_{R1} = 1000[lb \cdot mol]$ reactor's volume;

$q_{IA} = 100[\frac{lb \cdot mol}{h}]$ fresh feed volumetric flow rate of component A;

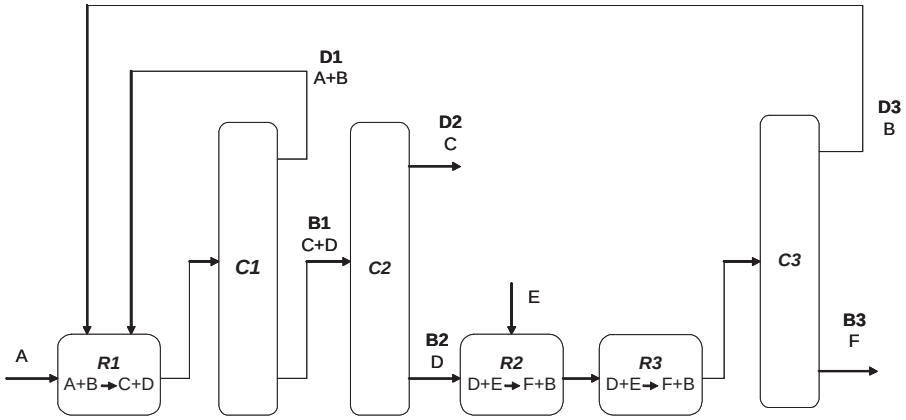

Figure 3.2: Flow diagram of the plant- ***R1, R2 R3*** are the reactors, ***C1, C2, C3*** are the distillation columns, **D1, D2, D3** are the top products and **B1, B2, B3** are the bottom products.

$q_{D1} = 80$ $[\frac{lb \cdot mol}{h}]$ volumetric flow rate of distillate **D1** from column ***C1***;

$q_{D3} = 141$ $[\frac{lb \cdot mol}{h}]$ volumetric flow rate of distillate **D3** from column ***C3***;

Reactor R2

$k_2 = 50[h^{-1}]$ reaction constant;

$V_{R2} = 850[lb \cdot mol]$ reactor's volume;

$q_{IE} = 99.5$ $[\frac{lb \cdot mol}{h}]$ fresh feed volumetric flow rate of component E;

Reactor R3

$k_3 = 141[h^{-1}]$ reaction constant;

$V_{R3} = 282[lb \cdot mol]$ reactor's volume;

Distillation column C1

$R_1 = 330[\frac{lb \cdot mol}{h}]$ reflux flow rate;

$V_1 = 410[\frac{lb \cdot mol}{h}]$ vapor flow rate;

$N_{a1} = 8$ number of trays of the enriching section;

$N_{e1} = 8$ number of trays of the stripping section;

$H_{r1} = 53.8$ $[lb \cdot mol]$ hold-up in the reboiler;

$H_{c1} = 33.6$ $[lb \cdot mol]$ hold-up in the condenser;

$H_{ar1} = 1.36$ $[lb \cdot mol]$ hold-up on the trays of the enriching section;

$H_{e1} = 1.63$ $[lb \cdot mol]$ hold-up on the trays of the stripping section;

$H_{al1} = 1.36\ [lb \cdot mol]$ hold-up on the feed tray.

Distillation column C2

$R_2 = 283[\frac{lb \cdot mol}{h}]$ reflux flow rate;

$V_2 = 385[\frac{lb \cdot mol}{h}]$ vapor flow rate;

$N_{a2} = 8$ number of trays of the enriching section;

$N_{e2} = 8$ number of trays of the stripping section;

$H_{r2} = 100\ [lb \cdot mol]$ hold-up in the reboiler;

$H_{c2} = 32.1\ [lb \cdot mol]$ hold-up in the condenser;

$H_{ar2} = 1.27\ [lb \cdot mol]$ hold-up on the trays of the enriching section;

$H_{e2} = 1.39\ [lb \cdot mol]$ hold-up on the trays of the stripping section;

$H_{al2} = 1.27\ [lb \cdot mol]$ hold-up on the feed tray.

Distillation column C3

$R_3 = 141[\frac{lb \cdot mol}{h}]$ reflux flow rate;

$V_3 = 282[\frac{lb \cdot mol}{h}]$ vapor flow rate;

$N_{a3} = 8$ number of trays of the enriching section;

$N_{e3} = 8$ number of trays of the stripping section;

$H_{r3} = 31.7\ [lb \cdot mol]$ hold-up in the reboiler;

$H_{c3} = 100\ [lb \cdot mol]$ hold-up in the condenser;

$H_{ar3} = 0.837\ [lb \cdot mol]$ hold-up on the trays of the enriching section;

$H_{e3} = 1.04\ [lb \cdot mol]$ hold-up on the trays of the stripping section;

$H_{al3} = 1.04\ [lb \cdot mol]$ hold-up on the feed tray.

3.2.2 Equilibrium conditions

Given the plant configuration previously described and the nominal inputs considered, the following equilibrium values have been computed.

Reactor R1

$x_{R1-A} = 0.2589$ molar fraction of component A inside the reactor;

$x_{R1-B} = 0.0356$ molar fraction of component B inside the reactor;

$x_{R1-C} = 0.2909$ molar fraction of component C inside the reactor;

$x_{R1-D} = 0.4146$ molar fraction of component D inside the reactor.

Reactor R2

$x_{R2-D} = 0.148$ molar fraction of component D inside the reactor;

$x_{R2-E} = 0.0152$ molar fraction of component E inside the reactor;

$x_{R2-F} = 0.4184$ molar fraction of component F inside the reactor;

$x_{R2-B} = 0.4184$ molar fraction of component B inside the reactor.

Reactor R3

$x_{R3-D} = 0.133$ molar fraction of component D inside the reactor;

$x_{R3-E} = 0.0006$ molar fraction of component E inside the reactor;

$x_{R3-F} = 0.4332$ molar fraction of component F inside the reactor;

$x_{R3-B} = 0.4332$ molar fraction of component B inside the reactor.

Column C1

$x_{C1-A(1)} = 0.0325$ molar fraction of component A in the reboiler;

$x_{C1-A(N_{p1})} = 0.9425$ molar fraction of component A in the condenser;

$x_{C1-B(1)} = 0.0319$ molar fraction of component B in the reboiler;

$x_{C1-B(N_{p1})} = 0.0468$ molar fraction of component B in the condenser;

$x_{C1-C(1)} = 0.3837$ molar fraction of component C in the reboiler;

$x_{C1-C(N_{p1})} = 0.0107$ molar fraction of component C in the condenser;

$x_{C1-D(1)} = 0.5519$ molar fraction of component D in the reboiler;

$x_{C1-D(N_{p1})} = 0$ molar fraction of component C in the condenser.

Column C2

$x_{C2-A(1)} = 0$ molar fraction of component A in the reboiler;

$x_{C2-A(N_{p1})} = 0.0769$ molar fraction of component A in the condenser;

$x_{C2-B(1)} = 0.0001$ molar fraction of component B in the reboiler;

$x_{C2-B(N_{p1})} = 0.0754$ molar fraction of component B in the condenser;

$x_{C2-C(1)} = 0.0593$ molar fraction of component C in the reboiler;

$x_{C2-C(N_{p1})} = 0.8274$ molar fraction of component C in the condenser;

$x_{C2-D(1)} = 0.9406$ molar fraction of component D in the reboiler;

$x_{C2-D(N_{p1})} = 0.0203$ molar fraction of component C in the condenser.

Column C3

$x_{C3-B(1)} = 0.$ molar fraction of component B in the reboiler;

$x_{C3-B(N_{p1})} = 0.7046$ molar fraction of component B in the condenser;

$x_{C3-D(1)} = 0.0075$ molar fraction of component D in the reboiler;

$x_{C3-D(N_{p1})} = 0.2208$ molar fraction of component D in the condenser;

$x_{C3-E(1)} = 0.001$ molar fraction of component E in the reboiler;

$x_{C3-E(N_{p1})} = 0.0003$ molar fraction of component E in the condenser;

$x_{C3-F(1)} = 0.9915$ molar fraction of component F in the reboiler;

$x_{C3-F(N_{p1})} = 0.0743$ molar fraction of component F in the condenser.

3.2.3 Linearized model

The model, in the configuration considered, has 183 state variables. The linearized model, computed through the numerical linearization procedures available in Matlab/Simulink has a very sparse dynamic matrix A, as shown in Figure 3.3 where the zero-non zero pattern of its elements is reported, reflecting the plant structure.

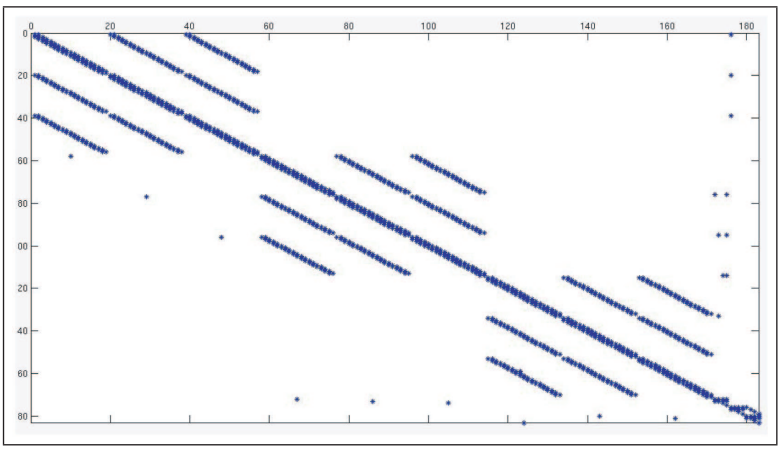

Figure 3.3: Matrix A of the linearized model

The eigenvalues of A are shown in Figure 3.4, as expected they are all inside the stability region of the complex plane.

The variables to be controlled are:

- x_{C1-At} molar fraction of component A at the top product of ***C1***;
- x_{C1-Db} molar fraction of component D at the bottom product of ***C1***;

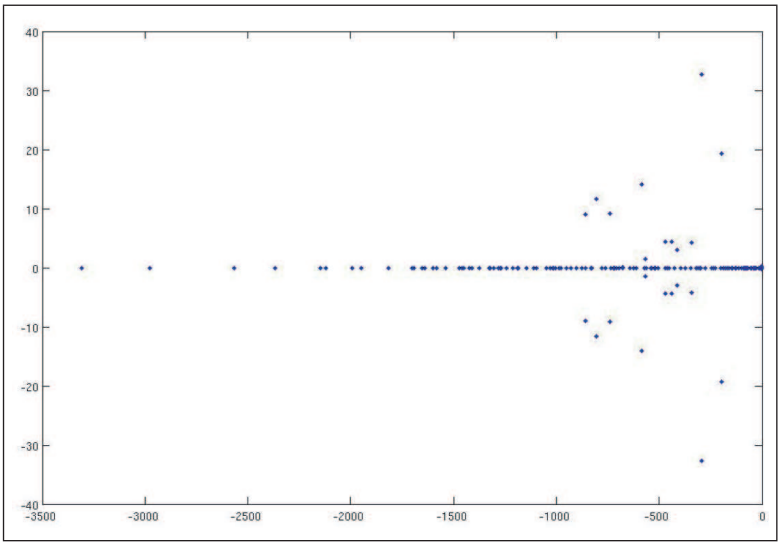

Figure 3.4: Eigenvalues of the linearized model

- x_{C2-Ct} molar fraction of component C at the top product of ***C1***;
- x_{C2-Db} molar fraction of component D at the bottom product of ***C2***;
- x_{C3-Bt} molar fraction of component B at the top product of ***C3***;
- x_{C3-Fb} molar fraction of component F at the bottom product of ***C3***.

The selected control variables are:

- R_1 reflux in ***C1***;
- V_1 vapor in ***C1***;
- R_2 reflux in ***C2***;
- V_2 vapor in ***C2***;
- R_3 reflux in ***C3***;
- V_3 vapor in ***C3***.

The step responses of this (6x6) system are shown in Figure 3.5; it is apparent that the system has strong interactions among the input and output variables, so that its control with a decentralized control structure is particularly difficult. These interactions will be examined in the following with some of the tools described in Chapter 2.

3.3 Analysis of the interactions of the linearized model

The linearized model of the plant is used to test some of the interaction measures and decomposition criteria described in the previous chapter. To this end, first note that the Relative Gain Array of the

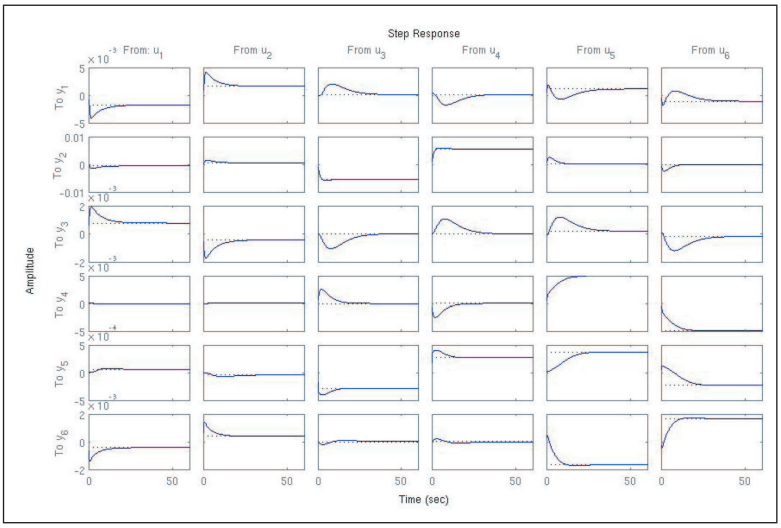

Figure 3.5: Step responses of the linearized model

linearized system is:

$$\begin{bmatrix} -1.366e-02 & 6.207e-03 & \boxed{3.574e-01} & \boxed{6.512e-01} & \boxed{8.093e-01} & -8.104e-01 \\ -9.690e-04 & 4.404e-04 & \boxed{5.999e-01} & \boxed{4.006e-01} & -4.034e-02 & \boxed{4.032e-02} \\ \boxed{2.730} & -1.730 & 1.170e-11 & -1.402e-11 & 8.442e-11 & -8.454e-11 \\ -5.370e-03 & 8.891e-03 & \boxed{4.027e-02} & -4.886e-02 & -5.267 & \boxed{6.272} \\ 3.542e-06 & -1.610e-06 & -1.695e-05 & 1.677e-05 & \boxed{2.654} & -1.654 \\ -1.710 & \boxed{2.714} & 2.424e-03 & -2.945e-03 & \boxed{2.844} & -2.848 \end{bmatrix} \tag{3.8}$$

where a frame has been placed to the elements which are positive and significantly different from zero. In view of this RGA, the possible input-output couplings are:

- Coupling $Co1$: $u_1 - y_3, u_2 - y_6, u_3 - y_1, u_4 - y_2, u_5 - y_5, u_6 - y_4$;
- Coupling $Co2$: $u_1 - y_3, u_2 - y_6, u_3 - y_2, u_4 - y_1, u_5 - y_5, u_6 - y_4$;
- Coupling $Co3$: $u_1 - y_3, u_2 - y_6, u_3 - y_4, u_4 - y_1, u_5 - y_5, u_6 - y_2$;
- Coupling $Co4$:$u_1 - y_3, u_2 - y_6, u_3 - y_4, u_4 - y_5, u_5 - y_1, u_6 - y_2$.

The Niederlinski index NI for each one of these pairings is:

- Coupling $Co1 \longrightarrow NI_1 = 2.58$;
- Coupling $Co2 \longrightarrow NI_2 = 4.7$;
- Coupling $Co3 \longrightarrow NI_3 = -1.09$;
- Coupling $Co4 \longrightarrow NI_4 = 5.09x10^{-4}$.

This result shows that the third set of couplings must be avoided, since it corresponds to a negative NI index. In order to select the best pairing among the remaining ones, it is possible to use the *Partial*

Relative Gain method. To this end, note that the coupling $u_1 - y_3$ is present in all the three possible configurations. Once this loop has been closed, one obtains

$$\Lambda(\bar{G}_{11})_{3xxxxx} = \begin{bmatrix} \boxed{0.0023} & \boxed{0.4040} & \boxed{0.5953} & \boxed{1.1286} & -1.1302 \\ \boxed{0.0002} & \boxed{0.5996} & \boxed{0.4003} & -0.0911 & \boxed{0.0911} \\ \boxed{0.0033} & \boxed{0.0401} & -0.0486 & -5.2681 & \boxed{6.2734} \\ -0.0000 & -0.0000 & 0.0000 & \boxed{2.5671} & -1.5671 \\ \boxed{0.9943} & -0.0436 & \boxed{0.0530} & \boxed{2.6635} & -2.6672 \end{bmatrix} \tag{3.9}$$

where the notation $\Lambda(\bar{G}_{11})_{3xxxxx}$ means that the input u_1 controls the output y_3, all the other loops being open. In view of the computed $\Lambda(\bar{G}_{11})_{3xxxxx}$ matrix, it is advisable to select the coupling $u_2 - y_6$, since the corresponding element is near to 1 (0.9943). By repeating the procedure to select the remaining coupling, one gets:

$$\Lambda(\bar{G}_{11})_{36xxxx} = \begin{bmatrix} -0.8610 & \boxed{1.8956} & \boxed{0.0539} & -0.0885 \\ \boxed{1.8748} & -0.8721 & \boxed{0.0042} & -0.0069 \\ -0.0138 & -0.0235 & -1.0916 & \boxed{2.1288} \\ -0.0000 & -0.0000 & \boxed{2.0334} & -1.0334 \end{bmatrix} \tag{3.10}$$

This matrix shows that the best coupling is $u_3 - y_2$ (which corresponds to the element (2,1) of $\Lambda(\bar{G}_{11})_{36xxxx}$). The next computed matrix is

$$\Lambda(\bar{G}_{11})_{362xxx} = \begin{bmatrix} \boxed{1.0008} & \boxed{0.0004} & -0.0012 \\ -0.0008 & -0.3527 & \boxed{1.3535} \\ 0.0000 & \boxed{1.3523} & -0.3523 \end{bmatrix} \tag{3.11}$$

which selects the coupling $u_4 - y_1$ (which corresponds to the element (2,1) of $\Lambda(\bar{G}_{11})_{362xxx}$). Finally, by inspection of

$$\Lambda(\bar{G}_{11})_{3621xx} = \begin{bmatrix} -0.0728 & \boxed{1.0728} \\ \boxed{1.0728} & -0.0728 \end{bmatrix} \tag{3.12}$$

the two remaining couplings $u_5 - y_5$ and $u_6 - y_4$ are chosen. In conclusion, the selected pairing is:

- $u_1 - y_3, u_2 - y_6, u_3 - y_2, u_4 - y_1, u_5 - y_5, u_6 - y_4$;

which corresponds to the physical variables:

- $R_1 \longmapsto c_{A1}$
- $V_1 \longmapsto c_{D1}$
- $R_2 \longmapsto c_{D2}$
- $V_2 \longmapsto c_{C2}$
- $R_3 \longmapsto c_{F3}$
- $V_3 \longmapsto c_{B3}$

This partitioning can then be used for the design of a decentralized control structure.

Chapter 4

Conclusions

In view of the results reported in this report, the future activity within Work Package 2 will be developed along the following research lines.

- Development of a mathematical formulation of the control design problem with MPC suitable for the description of many significant cases (decentralized, distributed and hierarchical systems, see Chapter 2).
- Extension of existing results to dynamically varying structures of the system under control, for example due to the inclusion or the removal of sensor or actuators (*plug and play* option).
- Analysis of the effects on the achievable performances when different communication protocols are used in distributed and hierarchical systems.
- Development of reduction/aggregation methods (see Chapter 3) tailored to the distributed framework here considered.

Chapter 5

Appendix

The Matlab/Simulink (version 7.0.1) simulator of the chemical benchmark example can be downloaded at the internet address

ftp://ftp.elet.polimi.it/users/Riccardo.Scattolini/

In order to use the simulator, proceed as follows:

- run the Matlab file *ChemBenchData.m* which initializes all the plant parameters and all the constant inputs. It is possible to edit this file and change anyone of these parameters.
- run the Simulink file *ChemBench.mdl*; note that in the simulator there is the block ”plots” where all the variables are plotted and stored.

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