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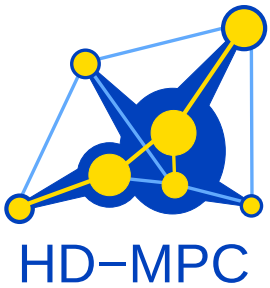


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## Executive Summary

The objective of this report is to provide a compact literature review on existing coordination mechanisms and to summarize results of the assessment of different coordination mechanisms. For this purpose we introduce the problem of coordination, on the hand by some practical motivation and on the other hand by some mathematical problem description. Coordination mechanisms are a crucial part of hierarchical and distributed model predictive control methods. The literature review provides an overview of the existing coordination methods: Many of them are closely related to each other and based on some price-driven coordination. Then we provide some short compact results of our own assessments. The results are analyzed regarding properties such as optimality and performance. Finally we give an overview of the results and discuss possible alternative coordination approaches.

# Chapter 1

# Introduction

Communication, cooperation, and coordination are the main properties that distinguish hierarchical and distributed model predictive control (MPC) methods from decentralized model predictive control methods. Decentralized MPC is considered to be the most important advanced control technology in large-scale systems, such as chemical plants or power plants, today. While the optimal control strategy for the entire plant, or a network of interconnected plants, would be to have a centralized model predictive controller, there are many challenges of this centralized method, that cannot be solved today, e.g.,

- If the size of the considered processes increases or the process time constants decrease, centralized MPC will demand tremendous computing power and thus will not be applicable for a real-time application [7, 30];
- In case of a spatially distributed system, e.g., a water supply network, communication among different subsystems of the global process might be limited;
- Maintainability of a centralized large-scale control structure is difficult [6];
- Reliability of a decentralized control structure might be better than the one of a centralized control structure [7].

Hence, today, the method of choice is to have decentralized MPCs[1], where each of the MPCs autonomously controls one part of the entire process, but without taking into account the interactions of the different subsystems. Hence, decentralized MPC will in general lead to a suboptimal control of the entire plant. Moreover, the neglection of interaction may also lead to reduced robustness of the control structure or even to instability. This can be seen in the following example: We shortly discuss the control problem of a plant with inputs $u \in \mathbb{R}^2$ and outputs $y \in \mathbb{R}^2$, which can be described by the linear transfer matrix:

$$G(s) = \begin{bmatrix} \frac{1}{s+1} & \frac{2}{s+6} \\ \frac{1}{s+3} & \frac{1}{s+1} \end{bmatrix}$$

If we use $K_1 = 20$ for the control of $G_{11}$ and $K_2 = 18$ for the control of $G_{22}$, we get a stable step response as depicted in Figure 1.1 on the left. However if the same controllers are applied to the entire system $G$, then we get an unstable step response as depicted in Figure 1.1 on the right.

[1] With a slight abuse of notation (note that MPC is an abbrevation of "model predictive control") we will use MPCs to denote "model predictive controllers" or "MPC-based controllers".

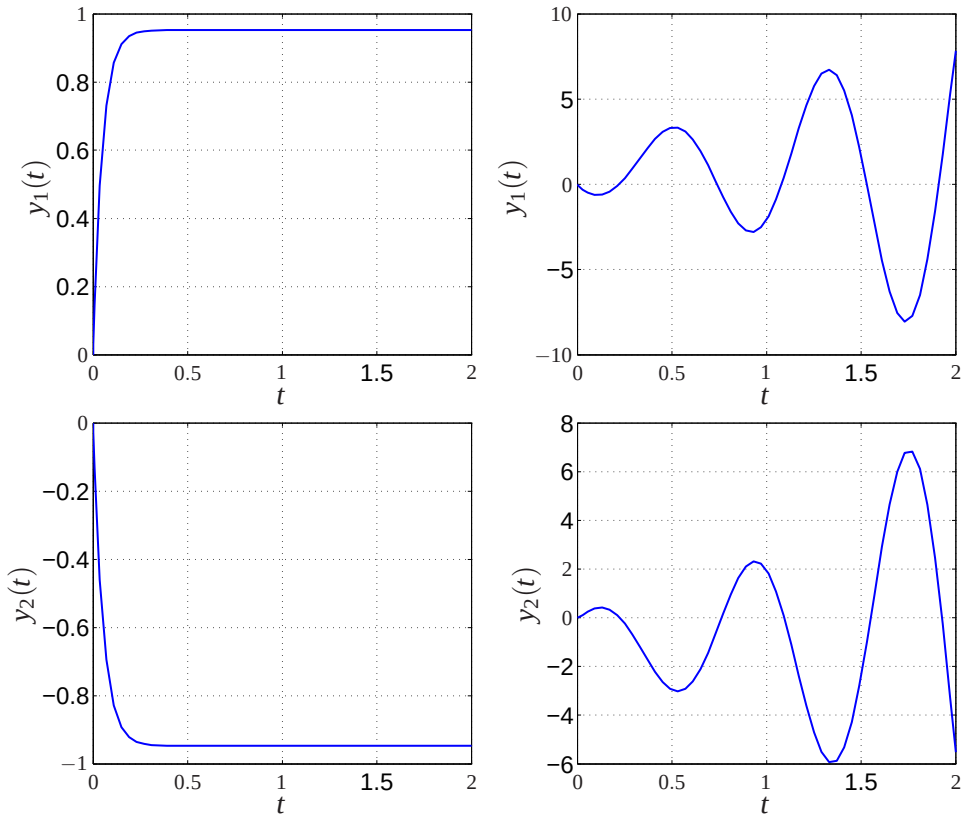



Figure 1.1: Instability due to interacting subsystems: On the left, there is the step response of decoupled subsystems. On the right, there is the step response for interacting subsystems.

Hence, the goal of hierarchical and distributed MPC is to have methods, combining the advantages of centralized and decentralized MPC:

- optimal control of the overall process,
- stability and robustness,
- low computational requirements,
- easy maintainability,
- reliability,
- low communication requirements.

Therefore, the approach of hierarchical and distributed MPC is to decompose the subordinate dynamical optimization problem of the MPC control into several smaller subproblems. However, in order to achieve stability, robustness, and optimality of the resulting accumulated solution, there has to be communication, cooperation and coordination between the different local (infimal) controllers, as it is depicted in Figure 1.2.

Before we formulate the problem in more detail, we first define the three different properties: communication, cooperation, and coordination:

- Communication means only a weak amount of interaction between different controllers. The controllers only share the information of the local subunit with other subunits. However, no information concerning overall optimality is spread. Hence, in general communication only will not result in optimal, stable, and robust plant operation [23].
- Cooperation means that compared to communication additional information is spread by local units in order to provide information, on how to achieve overall optimality. In MPC, this will also be referred to as distributed MPC.
- Coordination is almost the same as cooperation. However, there, the subproblems are coordinated by a supervisor, in order to achieve overall optimality. As there are two control layers, in MPC, this will also be referred to as hierarchical MPC.

The remainder of the report is organized as follows: Chapter 2 presents the mathematical problem of coordination. Chapters 3 and 4 provide a short literature overview for hierarchical and distributed coordination mechanisms. Then, in Chapter 5 we present some of our own assessment of coordination mechanisms. Finally, the conclusion (Chapter 6) summarizes the report and introduces possible future developments.

**Remark:** In the context of the HD-MPC project not only the tasks T3.1, T3.2 and T3.3 (*Hierarchical and distributed nonlinear MPC*, *Hierarchical and distributed robust nonlinear MPC* and *Coordination mechanisms*) cannot be handled independently, but also WP4 (*Optimisation methods for hierarchical and distributed MPC*) of the project is extremely closely related to the work to be done within WP3 (*Development of hierarchical and distributed MPC methods*). Hence, there will always be some overlapping parts within the related results.

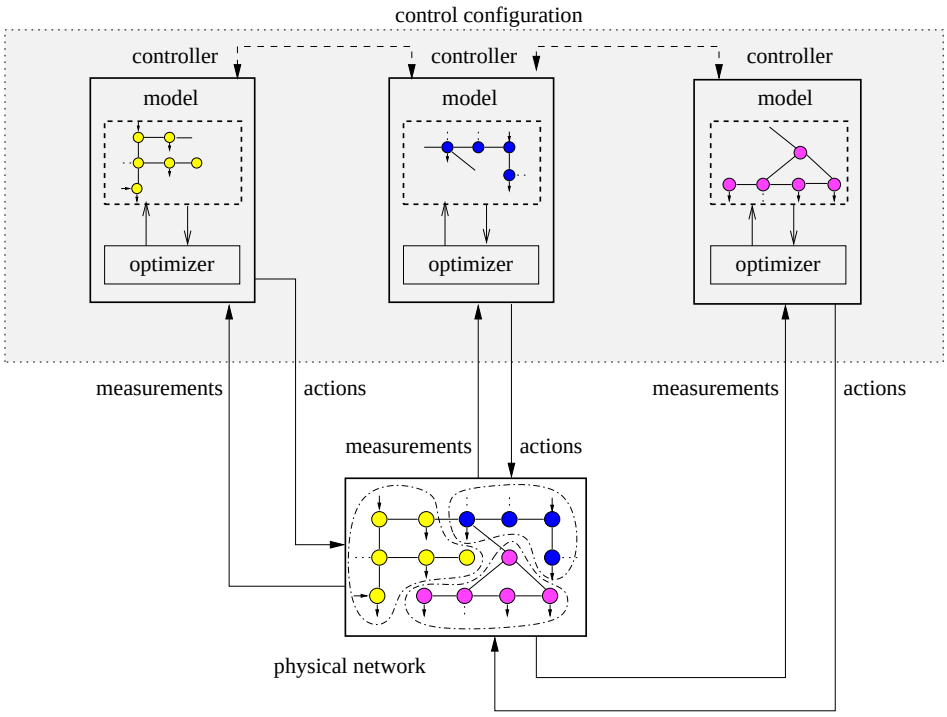



Figure 1.2: Illustration of distributed MPC control of a network

# Chapter 2

# Problem formulation

The idea of model predictive control (MPC) is to numerically solve a dynamic optimization problem at each sampling time. MPC uses a receding horizon approach, in which only the control signals for the first subsequent period of time are applied to the process; then the optimization problem has to be solved again for new initial conditions and again also only the first values for the manipulated variables are applied to the process. This method is repeated for each of the following sampling times of the control problem.

The same idea is to be implemented in hierarchical and distributed MPC. Thus, the basis for HD-MPC is an optimization problem, which has to be decomposed for different subsystems. As various versions of MPC are considered, e.g., linear and nonlinear MPC, discrete-time and continuous-time MPC, there exist also many different formulations of the optimization problem. Exemplarily in the following a quite general mathematical formulation of the control problem is introduced. We consider the global nonlinear optimal control problem:

$$\min_u \Phi(t,x,u,m), \tag{2.1a}$$

$$\text{w.r.t.} \quad \dot{x} = f(t,x,u,m), \quad x(0) = x_0, \tag{2.1b}$$

$$0 \leq g(t,x,u,m), \tag{2.1c}$$

$$0 \leq \pi(t_f, x(t_f), u(t_f), m(t_f)), \tag{2.1d}$$

$$m = H\,[x^T, u^T]^T, \tag{2.1e}$$

where $\Phi \in \mathbb{R}$ is the objective function, $t \in \mathbb{R}$ is the time, $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^p$ is the input vector, $m \in \mathbb{R}^q$ is the vector of coupling or interaction variables, i.e., the variables that represent the couplings between different subsystems, and $t_f$ is the final time of the optimization problem. The system dynamics are represented by (2.1b), the input and state constraints are summarized in (2.1c), and (2.1d) contains the constraints for the final time. The last equation (2.1e) describes the couplings within the system and is called the coupling constraint.

In case of the hierarchical and distributed MPC problem, the global optimization problem (2.1) has to be solved by the solution process of local problems for the $N$ subsystems, which are described

by the following local optimization problems:

$$\min_{u_i} \Phi_i(t, x_i, u_i, m_i), \tag{2.2a}$$

$$\text{w.r.t.} \quad \dot{x}_i = f_i(t, x_i, u_i, m_i), \quad x_i(0) = x_{i,0} \tag{2.2b}$$

$$0 \leq g_i(t, x_i, u_i, m_i) \tag{2.2c}$$

$$0 \leq \pi_i(t_f, x_i(t_f), u_i(t_f), m_i(t_f)), \tag{2.2d}$$

$$m_i = H_i [x^T, u^T]^T. \tag{2.2e}$$

Thereby $x_i$ is the local state vector, $u_i$ is the local input vector, and $m_i$ are the local coupling variables of the subsystem $i$. The local interaction variables depend on the global state and input variables. Hence, in case of hierarchical and distributed MPC one task of coordination or cooperation is to provide meaningful information on how to derive the local interaction variables.

It is important to keep in mind that the objective function is not necessarily additive, i.e., in general

$$\Phi(t, x, u, m) \neq \sum_{i=1}^{N} \Phi_i(t, x_i, u_i, m_i) \ . \tag{2.3}$$

Furthermore, the set of state variables $\{x_i | i = 1, \ldots, N\}$ does not necessarily have to be a disjunct set. However, usually the objective function $\Phi$ is assumed to be additive and the set of state variables is assumed to be disjunct.

The second task of coordination and cooperation is to modify the local optimization problems (2.2), such that the accumulated result $\{u_i^{\text{opt}} | i = 1, \ldots, N\}$ of the local optimization problems (2.2) is the same as the result $u^{\text{opt}}$ of the global optimization problem:

$$u^{\text{opt}} \stackrel{!}{=} \left[ \left(u_1^{\text{opt}}\right)^T, \left(u_2^{\text{opt}}\right)^T, \ldots, \left(u_N^{\text{opt}}\right)^T \right]^T \tag{2.4}$$

For this purpose, the local optimization problems (2.2) can be adapted in different ways: Coordination can be achieved by the intervention of the subsystems using coordination parameters, which can be divided into two subsets [14]: On the one hand the submodels can be modified, which is called model coordination; on the other hand, in goal coordination the objective function of the local problems are modified.

# Chapter 3

# Coordination mechanism in hierarchical model predictive control schemes

Here, we discuss two different coordination mechanisms for hierarchical MPC schemes, while in the following chapter we discuss coordination in distributed MPC. For an extended overview of HD-MPC methods, which are all related to coordination, see the review article [24] and the report [26].

## 3.1 Mixed method

This MPC method used in interconnected systems with different temporal dynamics [3], large scale systems [1] and systems with delays [2] is based on the local optimization of several defined cost functionals subject to the prediction of interactions among subsystems done by a coordinator layer. The method was firstly developed in 60's by Yasuhiko Takahara.
Let a discrete-time process be decomposed in $M$ subsystems as (3.1) shows, where $Z_i(k)$ are the interconnection relation at time $k$. Equation (3.2) shows the global cost function to optimize.

$$X_i(k+1) = A_i X_i(k) + B_i U_i(k) + C_i Z_i(k) \tag{3.1a}$$

$$Z_i(k) = \sum_{j=1}^{M} L_{ij} X_j(k) + K_{ij} U_j(k) \tag{3.1b}$$

$$J(U) = \sum_{i=1}^{M} \left\{ \frac{1}{2} X_i^T(N) P_i X_i(N) + \sum_{k=1}^{N-1} \left[ X_i^T(k) Q_i X_i(k) + U_i^T(k) R_i U_i(k) \right] \right\} \tag{3.2}$$

For this restricted optimization problem a solution method by using Lagrange multipliers is proposed. A coordinator layer is responsible to compute the interaction variables $Z_i(k)$ and a group of Lagrange multipliers $\beta_i(k)$ corresponding to the restrictions in (3.1b). Once the coordinator computes $Z_i(k)$ and $\beta_i(k)$, the global optimization problem defined by (3.1) and (3.2) can be decomposed in $M$ local optimization as (3.3) shows:

$$\min_{\{X_i, U_i\}} \Bigg\{ \sum_{k=1}^{N-1} (X_i^T(k) Q_i X_i(k) + U_i^T(k) R_i U_i(k) + \beta_i^{oT}(k) Z_i^{oT}(k) - \sum_{j=1}^{M} \beta_i^{oT}(k) \left[ L_{ij} X_j(k) + K_{ij} U_j(k) \right]) + \frac{1}{2} X_i^T(N) P_i X_i(N) \Bigg\} \tag{3.3}$$

where the superscript $*^{oT}$ refers to a value computed by the coordinator layer. The coordinator layer uses the first derivative criteria to compute the optimum for the coordination variables and its Lagrange multipliers ($Z_i(k)$ and $\beta_i(k)$).

This method solves $M$ local optimization problems in a parallel way, and they are harmonized by a recursive prediction of the coordination variables. The convergence was proved by A. Titli in 70's. Due to the parallel features and fast convergence reported in [3], this method becomes very interesting for large-scale systems.

## 3.2 Price-driven coordination

The coordination mechanism is based on the solution of several optimization problems. A total plant optimization problem described in (3.4) can be partitioned into several subproblems corresponding to controlled subsystems called unit-based MPCs. These subsystems are related by interactions that were not taken into account in the control design, i.e., they are in a decentralized structure:

$$\begin{aligned}
\min_{X_{set},U_{set}} \; & z_1 = (Y_{set}(k)-Y^*)^T Q_y (Y_{set}(k)-Y^*) + (U_{set}(k)-U^*)^T Q_u (U_{set}(k)-U^*) \\
& + C_Y (Y_{set}(k)-Y^*) + C_U (U_{set}(k)-U^*) + \eta^T C_\eta^T C_\eta \eta \\
\text{s.t.} \quad & \\
& Y_{set}(k) = \overline{\mathbb{A}} U_{set}(k) + E(k) + D(K) \\
& D(k) = D(k-1) + \Sigma(k) \\
& Y_{min} - \eta \leq Y_{set} \leq Y_{max} + \eta \\
& U_{min} \leq U_{set} \leq U_{max} \\
& \eta \geq 0 \\
& E(k) = \underline{\mathbb{A}} U_{set}(k)
\end{aligned} \tag{3.4}$$

The algorithm to solve the optimization problem above is based on the adding of a price that penalizes the interaction term $e(k)$ in the cost function as in equation (3.5) below. Then several subproblems (as many as there are unit-based MPCs in the total plant) are solved in each iteration as in equation (3.5) below. The prices for each subsystems are actualized at each iteration until they converge to an equilibrium value.

$$\begin{aligned}
\min_{x_{set},u_{set}} \; & z_2 = (y_{set}(k)-y^*)^T Q_y (y_{set}(k)-y^*) + (u_{set}(k)-u^*)^T Q_u (u_{set}(k)-u^*) \\
& + C_y (y_{set}(k)-y^*) + C_y (u_{set}(k)-u^*) + \eta^T C_\eta^T C_\eta \eta - \mathbf{p}^T \mathbf{e}(k) \\
\text{s.t.} \quad & \\
& y_{set}(k) - \mathbf{K}\mathbf{u}_{set}(k) = e(k) + d(K) \\
& d(k) = d(k-1) + \delta(k) \\
& y_{min} - \varepsilon \leq y_{set} \leq y_{max} + \varepsilon \\
& u_{min} \leq u_{set} \leq u_{max} \\
& \varepsilon \geq 0
\end{aligned} \tag{3.5}$$

In equations (3.4) and (3.5) upper case letters have been used to represent a set of all the variables in the whole plant and the lowercase letters have been used to represent the variables in a unit-based

MPC. Finally the algorithm to upgrade the price is defined as

$$\Delta(\mathbf{p}) = \mathbf{e}_i - \sum_{j=1}^{N} \mathbf{K}_{ij}\mathbf{u}_j, \qquad i = 1,...,N,\ j \neq i \tag{3.6}$$

This method has an important drawback that it requires a model of the entire plant were the interactions among unit-based MPCs are included. When the problem becomes large, likely a model of this type cannot be available.

# Chapter 4

# Distributed coordination methods: Literature review

In this chapter a literature review about coordination mechanisms in distributed control schemes is presented. For each approach described in this chapter, a brief description of the proposed method emphasizing in the coordination mechanisms is shown. From the description presented in the following sections it is possible to see that the base of the coordination mechanisms in distributed MPC schemes is the information exchange. This is the main way to confront the problem of coupling among subsystems in this control scheme and the main mechanism to carry out the cooperation.

## 4.1 Broadcast coordination

In the approach proposed in [5], the main objective is to achieve some degree of coordination among agents that are solving MPC problems with locally relevant variables, costs, and constraints without solving a centralized MPC problem. Also it is assumed that the connectivity of the communication network is sufficient for the agents to obtain information regarding all the variables that appear in their local problems. Such coordination schemes are useful when the local optimization problems are much smaller in size than a centralized problem, as in network control applications, but they are determined by the information structure, i.e., the connectivity and capacity of the interagent communication network.

Here, the coordination mechanism used was to send a broadcast to the neighbors with the results of the last iteration, and to receive from them the same information to adjust the Lagrange multipliers associated with the exchanged variables (interconnection variables).

## 4.2 Coordination based on prediction interchange

Similarly to the coordination mechanism depicted in the previous section, in [11] a distributed MPC scheme with stability constraints is proposed. Each controller is coordinated with the others by exchanging their predictions. In this approach there is no centralized coordinator. This is an interesting feature because it provides robustness to the decentralized control scheme even if communication failures are present.

To carry out the proposed method local MPCs are used. The control actions are computed based on local state measurements. The controllers exchange information about their measurements and

predictions and incorporate this information in their local computations.

In this case, the model used to describe the local agent dynamics is given by

$$x_i(k+1) = F_i(x_i(k), u_i(k), v_i(k), w_i(k)) \tag{4.1}$$

where $x_i$ denotes the states of all neighbor subsystems, $u_i$ denotes de local control inputs, $v_i$ denotes the neighbor predictions, and $w_i$ denotes the disturbances. Thus the local optimization problem becomes

$$\min_{x_i(k),u_i(k)} J_{local}(x_i(k), u_i(k)) \tag{4.2}$$

In this work, the coordination is given through the exchange of local measurements and predictions, and by taken it into account in the prediction model of each subsystem inside a neighborhood.

## 4.3 Distributed model predictive control as a game with coupled constraints

In [29], formally the benefits of cooperation among subsystems in a distributed MPC scheme are analyzed. In this work, the cooperation is carried out by local agents taking into account the objectives of the others. Also for each agent $p$ it is assumed that there exists a controller $u_p = k_p x_p$. Thus the MPC problem is formulated in a distributed fashion as

$$\min_{u_p(k),\widetilde{u}_p(k)} J_p(x_p(k), u_p(k)) + \sum_{q\in\mathscr{N}_p} J_q(x_q(k), \widetilde{u}_q(k)) \tag{4.3}$$

subject to the discrete solution of the system dynamics over the prediction horizon, and the subsystem operational constraints, where $\mathscr{N}_p$ is the set of neighbors of agent $p$. In this case, it was assumed that the neighbor subsystems are coupled by the constraints modeled as

$$\sum_{q\in\mathscr{N}_p} E_{cq}x_q + F_{cq}k_q x_q \in Z_c, \quad \forall c \in C_p, \tag{4.4}$$

where $C_p$ denotes the set of constraints involving the subsystem $p$.

In the objective function, the term $J_q(x_q(k), \widetilde{u}_q(k))$ is associated with the objectives of the neighbor subsystems. Moreover, the optimization is solved taking into account the neighbor control inputs $\widetilde{u}_q(k)$, however its values are not communicated to the others.

To assure optimality, in this approach only one subsystem at each sample time solves its local optimization problem, thus the computed control inputs converge at each time to a Nash equilibrium point. This is important because, from the point of view of game theory, the Nash equilibrium point is the optimal solution of a non-cooperative game. To solve the optimization problem in serial way, a central agent is required to assign the sequence of solution.

## 4.4 Dynamic dual decomposition for distributed control

In the same way, in [22] the Lagrange multipliers are conceived as prices in a market mechanisms serving to achieve mutual agreements among the subproblems (agents). Based on the previous interpretation of Lagrange multipliers, in this reference dynamic price mechanisms were used for decomposition and distribution of the optimization of the control systems. To develop the topics presented

before the dual decomposition and saddle algorithm is introduced and applied to optimal control problems.

Also in [21] ideas from game theory and economics, and its role in decentralized controller design in complex engineering systems are presented. [21] starts with presenting ideas and notions of game theory such as the definition of game, payoff function, Nash equilibrium point, and team problems. Then, a team problem with a graph structure is studied, and finally a team problem with strategies with global impact is introduced.

An application of the concepts and ideas presented in [21] is presented in [20]. In this reference an optimal control, based on team theory, for a system with stochastic uncertainties and information delays is proposed, considering a linear quadratic stochastic control problem. The problem involves several different controllers acting as a team, but with access to different measurements.

To formulate the optimization problem as a cooperative game, the subsystems representation used is as follows

$$\begin{aligned} x_i(t+1) &= A_{i1}x_1(t)+\ldots+A_{ii}x_i(t)+\ldots+A_{in}x_n(t) \\ &\quad +B_{i1}u_1(t)+\ldots+B_{ii}u_i(t)+\ldots+B_{in}u_n(t) \end{aligned} \tag{4.5}$$

Let us, for all $i \neq j$, define $v_{ij}$ as the opinion of subsystem $i$ about the value of the state variables of subsystem $j$, $w_{ij}$ as the opinion of subsystem $i$ about the value of the input variables of subsystem $j$, $p_{ij}$ as the price associated to the influence of subsystem $i$ in the state variables of subsystem $j$ , and $q_{ij}$ as the price associated to the influence of subsystem $i$ in input variables of subsystem $j$. Then the subsystem representation becomes

$$\begin{aligned} x_i(t+1) &= A_{i1}v_{i1}(t)+\ldots+A_{ii}x_i(t)+\ldots+A_{in}v_{in}(t) \\ &\quad +B_{i1}w_{i1}(t)+\ldots+B_{ii}u_i(t)+\ldots+B_{in}w_{in}(t) \end{aligned} \tag{4.6}$$

From the system representation (4.6), using dual decomposition and team theory, the optimization problem can be rewritten as follows [22]:

$$\begin{aligned} &\max_{p_{ij},q_{ij}} \min_{x_i(t),u_i(t)} \sum_{k=0}^{N-1} l_i[x_i(t+k),u_i(t+k)]+F_i+G_i \\ \text{s.t.} \quad &x_i(t+1)=A_{ii}x_i(t)+B_{ii}u_i(t) \\ &\text{operational constraints} \end{aligned} \tag{4.7}$$

where

$$\begin{aligned} F_i(x_i(t),v) &= 2\sum_{j\in\Psi(i)}[p_{ij}]^T(x_i-v_{ij}) \\ G_i(x_i(t),w) &= 2\sum_{j\in\Psi(i)}[q_{ij}]^T(u_i-w_{ij}) \end{aligned}$$

In (4.7), the strategy of each node is $\mu_i=(x_i(t),u_i(t))$ and its payoff function is

$$l_i(x_i(t+k),u_i(t+k))+F_i(x_i(t+k),v)+G_i(x_i(t+k),w) \quad ,$$

where $l_i(x_i(t+k),u_i(t+k))$ is its own cost, and $F_i(x_i(t+k),v)$ $+G_i(x_i(t+k),w)$ is the price that the $i$th subsystem pays to its neighbors, or receives from them, for influencing their behavior. This is a game with $n$ players and a number of "market makers", who adjust the price variables $p_{ij}$ and $q_{ij}$. The

Nash equilibrium of this game corresponds to a global optimum of the original optimization problem [22].

The optimization problem formulation (4.7), has an advantage that no global models are required to find the optimal solution. Moreover, it is more general than the formulation proposed in [22, 21], because it includes the influence of the neighbors inputs into the behavior of the *i*th subsystem. With the inclusion of the influence of the inputs, it is possible to avoid the troubles reported in [34] about suboptimal convergence and its related unstable behavior.

## 4.5 Non-cooperative game approach to distributed model predictive control

In [32, 31], a formulation of distributed MPC as a non-cooperative game is proposed. In this approach, the negotiation among agents is included as a step of the game solution [16]. It gives to each agent the possibility of deciding whether it takes into account the neighbors' information in its local optimization problem, depending on its local payoff function value.

A distributed MPC problem can be viewed as a set $G = \{N, \{\mu_i\}_{i\in N}, \{v_i\}_{i\in N}\}$ where $N$ is a finite set of $n$ subsystems, $\mu_i$ is a finite set of control actions of the $i$-th subsystem, and $v_i$ is the performance index associated with the control actions of the $i$-th player. Thus the distributed MPC problem can be formulated as a non-cooperative game as follows:

$$\begin{aligned} & \max_{\mu_i}\{\sum_{i=1}^{n}\theta_i^N \log[d_i^N - v_i(x_i,\mu_i)]\} \\ \text{s.t. } & d_i^N \geq v_i(x_i,\mu_i) \\ & \mu_i \in \Delta(\mu_i) \\ & x_i(k+1) = \sum_{j=1}^{n}[A_{ij}x_j(k) + B_{ij}\mu_j(k)] \\ & y_i(k+1) = C_i x_i(k+1) + D_i\mu_i(k) \\ & \text{operational constraints} \end{aligned} \tag{4.8}$$

or in distributed form

$$\begin{aligned} & \max_{\mu_i}\{\theta_i^N \log[d_i^N - v_i(x_i,\mu_i)]\} \\ \text{s.t. } & d_i^N \geq v_i(x_i,\mu_i) \\ & \mu_i \in \Delta(\mu_i) \\ & x_i(k+1) = \sum_{j=1}^{n}[A_{ij}x_j(k) + B_{ij}\mu_j(k)] \\ & y_i(k+1) = C_i x_i(k+1) + D_i\mu_i(k) \\ & \text{operational constraints} \end{aligned} \tag{4.9}$$

where $d_i^N$ can be defined as the maximum deviation from the goal allowed for the system or

computed as the optimal solution of the optimization problem

$$\begin{aligned} & \min_{\mu_i(k)} v_i[x_i(k), \mu_i(k)] \\ \text{s.t. } & x_i(k+1) = A_i x_i(k) + B_i \mu_i(k) \\ & y_i(k+1) = C_i x_i(k+1) + D_i \mu_i(k) \\ & \text{operational constraints} \end{aligned} \tag{4.10}$$

The MPC problem formulation shown in (4.9) allows to quantify the effects of taking into account the neighbors' information and the benefits for the system when the subsystems cooperate. It can be seen in (4.9) that only if the cooperation among subsystems gives a best payoff, the subsystems cooperate, otherwise each subsystem works alone.

# Chapter 5

# Assessment of coordination mechanisms

In the following sections, we briefly summarize the results of the assessment of different coordination mechanisms.

## 5.1 Communication-based distributed MPC

Venkat et al. have already assessed communication-based MPC for discrete-time linear time-invariant systems [35, 23]. In a first example the communication-based MPC method not only resulted in a suboptimal control performance. Moreover, the resulting MPC leads to an unstable behaviour of the example system. However, for a second example, the communication-based MPC method leads to a stable system behaviour with a system performance close to the optimal control, i.e., with huge improvement compared to decentralized MPC.

### 5.1.1 Communication-based MPC for event-driven continuous-time systems

In [28] we consider a communication-based distributed MPC approach for an event-driven continuous-time model, namely a baggage handling system. The baggage handling system of an airport is one of the most important factors that determine the airport's efficiency and reliability. State-of-the-art baggage handling systems transport luggage in an automated way using destination coded vehicles (DCVs). These vehicles transport the bags at high speeds on a "mini" railway network. The considered DCV-based baggage handling system is sketched in Figure 5.1. This system operates as follows: given a dynamic demand of bags (identified by their unique code) and a buffer of empty DCVs for each loading station, together with the network of tracks, the optimal route of each DCV (from a given loading station to the corresponding unloading station) has to be computed subject to operational and safety constraints such that the performance of the system is optimized.

The considered track network has $S$ junctions $S_1$, $S_2$,..., $S_S$, while each junction has at most 2 incoming links and at most 2 outgoing links, both indexed by $l \in \{0,1\}$. Each junction has a switch going into the junction and a switch going out of the junction.

There are five types of events that can occur:

- loading a new bag into the system
- unloading a bag that arrives at its end point
- updating the position of the switch-in

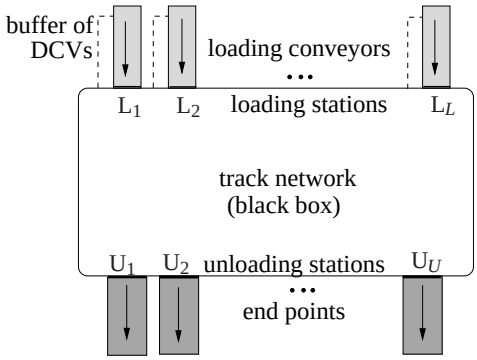



Figure 5.1: Baggage handling system using DCVs.

- updating the position of the switch-out
- updating the speed of a DCV.

The model of the baggage handling system is an event-driven one consisting of a continuous part describing the movement of the individual vehicles transporting the bags through the network, and of the discrete events listed above.

**Distributed model predictive control**: Here performance of the local, decentralized control is increased by implementing a *distributed* approach that uses additional communication and coordination between neighboring junctions. Data will be communicated between consecutive levels of influence. It is considered that the communication of the future actions is performed downstream.

The computation of the local control is performed according to the following algorithm where $\mathscr{K}$ is the largest level of influence assigned in the network.

**Algorithm: Distributed computation of local control**

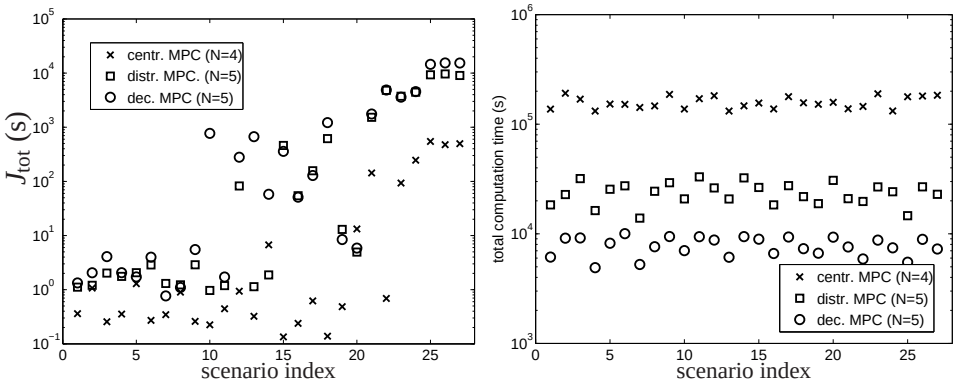
1: **for** $\kappa = 1$ to $\mathscr{K}$ **do**
2: compute in parallel local switching sequences for influence level $\kappa$ taking into account the control on influence level $\kappa - 1$
3: **end for**

Every time a bag crosses a junction the local control of all junctions is updated. Recall that the controllers of the junctions on level $\kappa$ have to wait for the completion of the computation of the switching sequences of the controllers on the previous level before starting to compute their future control action. Therefore, when comparing with decentralized MPC, such distributed MPC may improve the performance of the system, but at the cost of higher computation time due to the required synchronization and iteration in computing the control actions.

**Results:** To solve the optimization problems a *genetic* algorithm of the Matlab optimization toolbox has been chosen with multiple runs.

Based on simulations the proposed control methods could be compared for the given scenarios. Figure 5.2 shows the results obtained when using centralized, decentralized, and respectively distributed MPC for various scenarios.

Clearly the best performance of the system is obtained when using centralized switch control. However, centralized control becomes intractable in practice when the number of junctions is large due to the large computation time required. The simulations indicate that both decentralized MPC and




(a) closed loop performance (b) computation time

Figure 5.2: Comparison of the proposed control approaches.

distributed MPC offer a balanced trade-off between computation time and optimality. However, the results confirm that the communication of the intended control action between neighboring junction may increase the performance of the system, but at the cost of bigger computational effort.

## 5.2 Price-driven coordination methods

Price-driven coordination methods clearly dominate research of hierarchical and distributed MPC as well as distributed, large-scale optimization. While there also exist alterative approaches for price-driven coordination, e.g., in [15], the dual optimization method [12, 13] seems to be the most important method, today. We have assessed this method for different applications, linear and nonlinear, and also considered modified versions of dual optimization.

### 5.2.1 Dual optimization with an additional penalty term

In this application we consider water networks. To improve the operation of water systems the controllers that control different parts of the water network should cooperate and coordinate their local water management actions, and take into account predictions or forecasts of future rain fall, future droughts, future arrival of increased water flow via rivers, etc. (using various weather and hydrological sensors, and prediction models). Using distributed MPC more efficient flood and water management with less risks and less costs can be obtained. In [18, 19] we apply a particular distributed MPC scheme, recently proposed in [17], for improving the operation of a particular type of water systems, viz. irrigation canals.

**Coordination mechanism:** Overall optimality is achieved by a modification of the objective function of the local MPC units, i.e., there is a modified optimization problem:

$$\begin{aligned}\min_{\substack{\tilde{\mathbf{x}}_i(k+1),\tilde{\mathbf{u}}_i(k),\tilde{\mathbf{y}}_i(k),\\ \tilde{\mathbf{w}}_{\text{in},j_{i,1}i}(k),\ldots,\tilde{\mathbf{w}}_{\text{in},j_{i,m_i}i}(k),\\ \tilde{\mathbf{w}}_{\text{out},j_{i,1}i}(k),\ldots,\tilde{\mathbf{w}}_{\text{out},j_{i,m_i}i}(k)}} \quad & J_{\text{local},i}\left(\tilde{\mathbf{x}}_i(k+1),\tilde{\mathbf{u}}_i(k),\tilde{\mathbf{y}}_i(k)\right)\\ & + \sum_{j\in\mathcal{N}_i^{(s)}} J_{\text{inter},i}^{(s)}\left(\tilde{\mathbf{w}}_{\text{in},ji}(k),\tilde{\mathbf{w}}_{\text{out},ji}(k)\right),\end{aligned} \tag{5.1}$$

subject to the local dynamics of subnetwork $i$ over the horizon. In this, the additional objective

function $J_{\text{inter},i}$ at iteration $s$ is defined as

$$J^{(s)}_{\text{inter},i}\left(\tilde{\mathbf{w}}_{\text{in},ji}(k),\tilde{\mathbf{w}}_{\text{out},ji}(k)\right) = \begin{bmatrix}\tilde{\lambda}^{(s)}_{\text{in},ji}(k)\\ -\tilde{\lambda}^{(s)}_{\text{out},ij}(k)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\tilde{\mathbf{w}}_{\text{in},ji}(k)\\ \tilde{\mathbf{w}}_{\text{out},ji}(k)\end{bmatrix} + \frac{\gamma_{\text{c}}}{2}\left\|\begin{bmatrix}\tilde{\mathbf{w}}_{\text{in,prev},ij}(k)-\tilde{\mathbf{w}}_{\text{out},ji}(k)\\ \tilde{\mathbf{w}}_{\text{out,prev},ij}(k)-\tilde{\mathbf{w}}_{\text{in},ji}(k)\end{bmatrix}\right\|_2^2.$$

This additional objective function $J_{\text{inter},i}$ consists of two parts:

- The first term is well-known, from dual optimization: A product of Lagrange multipliers and the corresponding constraints.
- The second term is an additional penalty term for unsatisfied constraints.

The Lagrange multipliers are updated as in dual optimization:

$$\tilde{\lambda}^{(s+1)}_{\text{in},ji}(k) = \tilde{\lambda}^{(s)}_{\text{in},ji}(k) + \gamma_{\text{c}}\left(\tilde{\mathbf{w}}^{(s)}_{\text{in},ji}(k) - \tilde{\mathbf{w}}^{(s)}_{\text{out},ij}(k)\right).$$

**Model:** The method has been assessed for a system of irrigation channels. The dynamics of irrigation canals can be modeled in detail, e.g., using the Saint Venant equations [8] resulting in systems of highly-nonlinear partial differential-algebraic equations. However, similarly as in [33], the integral delay model [27] has been employed to model the dynamics of a canal reach. This model has shown to adequately capture relevant dynamics [27], and reduces computations required for simulation of the dynamics significantly.

The model describing how the level $h_i$ of the water in the canal reach $i$ changes from one control cycle $k$ to the next control cycle $k+1$ is given by:

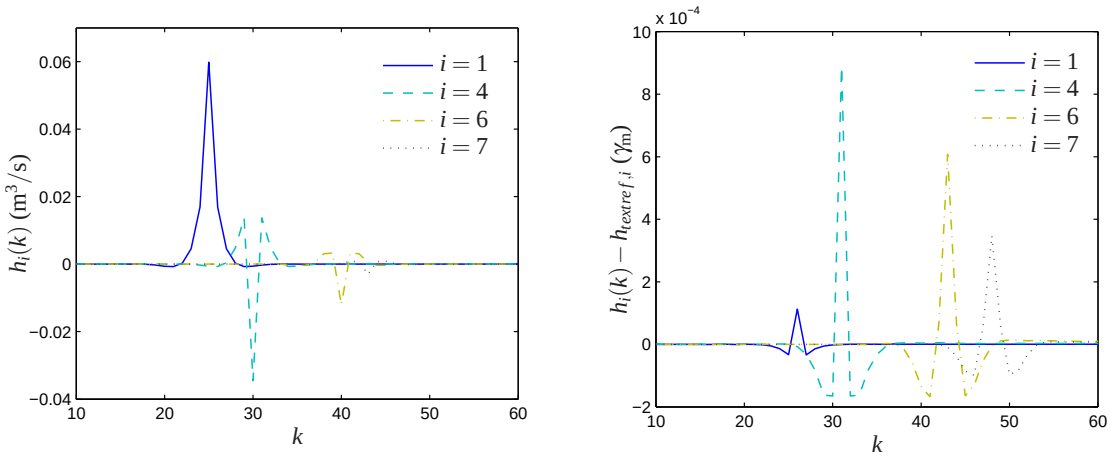
$$h_i(k+1) = h_i(k) + \frac{T_{\text{c}}}{c_i}q_{\text{in},i}(k-k_{\text{d},i}) - \frac{T_{\text{c}}}{c_i}q_{\text{out},i}(k) + \frac{T_{\text{c}}}{c_i}q_{\text{ext,in},i}(k) - \frac{T_{\text{c}}}{c_i}q_{\text{ext,out},i}(k). \tag{5.2}$$

Canal reaches are connected to one another. When two canal reaches are connected to each other, the inflow of one canal reach is equal to the outflow of the other. Hence, for neighboring reaches $i$ and $j$ this interconnection is given by

$$q_{\text{out},i}(k) = q_{\text{in},j}(k). \tag{5.3}$$

**Results:** Fig. 5.3(a) shows the changes in the set-points decided upon by the controllers. Fig. 5.3(b) shows the closed-loop evolution of the deviations of the water levels from the reference values. It can be seen that the inflow of canal reach 1 is increased right before the additional offtake increase takes place in order to prevent having a too low water level after the additional offtake. It can also be observed that the deviations of the water levels after the offtake increase are minimal due to the changes in the set-points. We observe that after about 25 control cycles the set-points settle at a constant value, while the deviations of the water levels from the references are minimal, and that thus the controllers have performed their tasks adequately.

The costs computed over the full simulation using the distributed MPC scheme are $1832.10^{-7}$. A centralized MPC controller based on the same objectives obtains costs over the full simulation of $1831.10^{-7}$. This difference in performance is negligible, and hence, in this case in which the assumptions made are valid, indeed, the distributed controllers have achieved a performance comparable to the performance obtained by a centralized MPC controller.




(a) (b)

Figure 5.3: Evolution for four representative canal reaches of (a) set-points and (b) deviation of the water levels from reference values.

### 5.2.2 Dual optimization with Han's method

In another paper [9] we have assessed the dual optimization approach, where the dual problem has been using solved Han's method [10]. The following discrete-time, linear time-invariant model has been considered:

$$x^i(k+1) = \sum_{j\in\mathcal{N}^i} A^{ij}x^j(k) + B^{ij}u^j(k), \tag{5.4}$$

An important part of this assessment has been the quite general type of constraints, i.e.,

$$\sum_{j\in\mathcal{N}^i}\sum_{k=0}^{N-1} D^{ij}(k)x^j(k) + E^{ij}(k)u^j(k) = c_{\text{eq}} \tag{5.5}$$

$$\sum_{j\in\mathcal{N}^i}\sum_{k=0}^{N-1} \bar{D}^{ij}(k)x^j(k) + \bar{E}^{ij}(k)u^j(k) \leq \bar{c}_{\text{ineq}} \tag{5.6}$$

For the optimization within the MPC problem a decoupled and convex quadratic cost function

$$J = \sum_{i=1}^{M}\sum_{k=0}^{N-1}\left(\left(u^i(k)\right)^T R_i u^i(k) + \left(x^i(k+1)\right)^T Q_i x^i(k+1)\right) \tag{5.7}$$

is assumed with positive definite weights $R_i, Q_i, \forall i$. Hence, the problem can be reformulated in a compact form as:

$$\begin{aligned} \min_{\mathbf{x}} \quad & \mathbf{x}^T H \mathbf{x} \\ \text{s.t.} \quad & a_l^T \mathbf{x} = b_l, \quad l = 1,\ldots,n_{\text{eq}} \\ & a_l^T \mathbf{x} \leq b_l, \quad l = n_{\text{eq}}+1,\ldots,s. \end{aligned} \tag{5.8}$$

**Coordination method:** The main idea is to use Han's algorithm [10] to solve the dual problem of the centralized optimization using parallel computations in an iterative scheme.

*Han's algorithm for general convex problems*: The class of optimization problems tackled by Han's algorithm is the following:

$$\begin{aligned} \min_{\mathbf{x}} \quad & q(\mathbf{x}) \\ \text{s.t.} \quad & \mathbf{x} \in C \triangleq C_1 \cap \cdots \cap C_s \end{aligned} \tag{5.9}$$

where $C_1, \cdots, C_s$ are closed convex sets and $C \neq \varnothing$.

The algorithm is an iterative procedure. We use $p$ as iteration counter of the algorithm. We use a superscript $(p)$ to denote the values of variables computed at iteration $p$.

Let $\alpha$ be a sufficiently large number[1] and define $\mathbf{y}^{(0)} = \mathbf{y}_1^{(0)} = \cdots = \mathbf{y}_s^{(0)} = 0$, with $\mathbf{y}^{(0)}, \mathbf{y}_l^{(0)} \in \mathbb{R}^{n_{\mathbf{x}}}, l = 1, \ldots, s$, and $\mathbf{x}^{(0)} = \nabla q^* \left(\mathbf{y}^{(0)}\right)$ with $q^*$ being the conjugate function[2] of $q$. For $p = 1, 2, \ldots$, we perform the following computations:

1) For $l = 1, \ldots, s$, find $\mathbf{z}_l^{(p)}$ that solves

$$\begin{aligned} \min_{\mathbf{z}} \quad & \frac{1}{2} \| \mathbf{z} + \alpha \mathbf{y}_l^{(p-1)} - \mathbf{x}^{(p-1)} \|_2^2 \\ \text{s.t.} \quad & \mathbf{z} \in C_l \end{aligned} \tag{5.10}$$

2) Assign

$$\mathbf{y}_l^{(p)} = \mathbf{y}_l^{(p-1)} + (1/\alpha) \left( \mathbf{z}_l^{(p)} - \mathbf{x}^{(p-1)} \right) \tag{5.11}$$

3) Set $\mathbf{y}^{(p)} = \mathbf{y}_1^{(p)} + \cdots + \mathbf{y}_s^{(p)}$

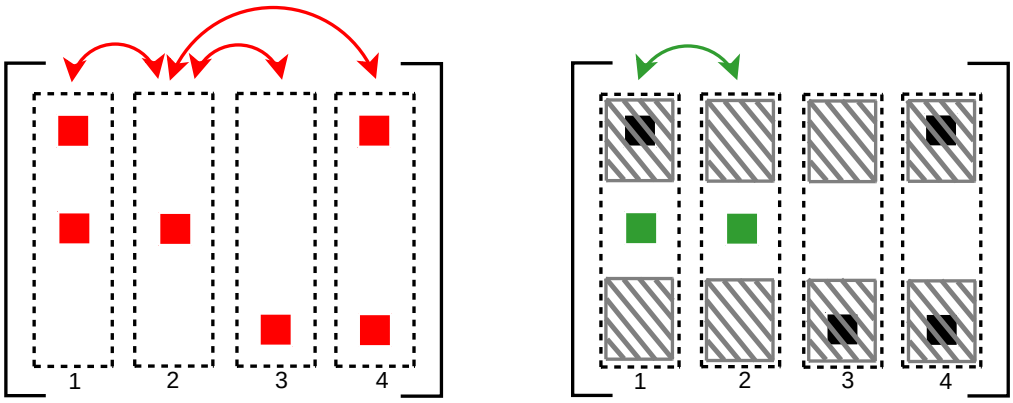
4) Compute

$$\mathbf{x}^{(p)} = \nabla q^* \left( \mathbf{y}^{(p)} \right) \tag{5.12}$$

*Distributed version of Han's algorithm with regional coordination method*: Han's algorithm involves calculation of the global variables, therefor a global coordination method is required. By exploiting the structure of problem (5.8), we can implement a distributed version of Han's algorithm that requires a regional coordination method, in which each subsystem coordinates the computations of several variables and thus only needs to communicate with its neighbors.

This idea is illustrated in Figure 5.4, with a simple system consisting of 4 subsystems and the coupling matrix that shows how subsystems are coupled via their variables (boxes on a same row illustrate the variables that are coupled in one constraint). In the centralized coordination version, a subsystem has to communicate with all other subsystems in order to compute the updates of global variables. For the distributed coordination version, each subsystem only communicates with the other subsystems of which the variables are necessary for computing the updates of its local variables.

**Model:** For the assessment of the method, here a model of coupled oscillators has been used. The system consists of $M$ oscillators that can move only along the vertical axis, and that are coupled by springs that connect each oscillator with its two closest neighbors. An exogenous vertical force will




(a) (b)

Figure 5.4: Communication links of the 2nd subsystem with (a) the centralized coordination version and (b) the distributed coordination version of Han's algorithm for an example 4-subsystem problem.

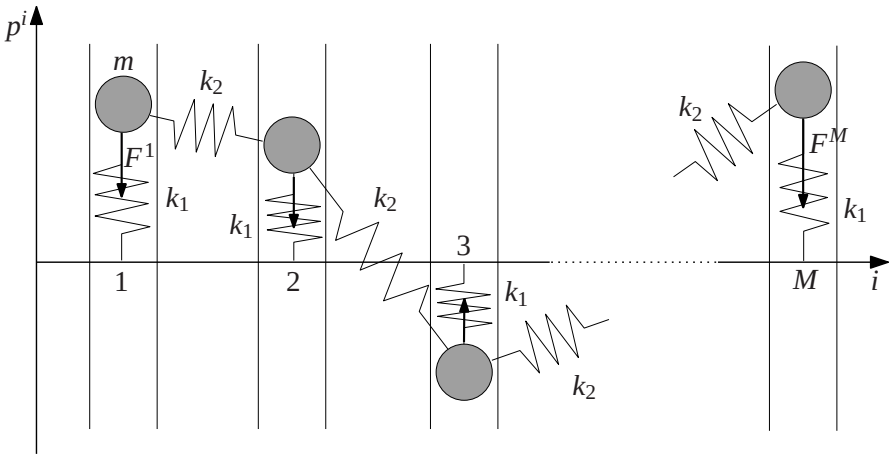



Figure 5.5: Setup with coupled oscillators

be used as the control input for each oscillator. The setup is shown in Figure 5.5. Each oscillator is considered as one subsystem. Let the superscript $i$ denote the index of the oscillators. The continuous-time dynamics equation of oscillator $i$ is then defined as

$$ma^i(t) = k_1 p^i(t) - f_s v^i(t) + k_2(p^{i-1}(t) - p^i(t)) + k_2(p^{i+1}(t) - p^i(t)) + F^i(t), \tag{5.13}$$

where $p^i(t)$, $v^i(t)$, and $a^i(t)$ denote the position, velocity, and acceleration of oscillator $i$ at time $t$, respectively. The control force exerted at oscillator $i$ is $F^i(t)$. The system parameters are $k_1$, $k_2$, $m$, and $f_s$, representing respectively the stiffness of the vertical spring at each oscillator, the stiffness of the springs that connect the oscillators, the mass of each oscillator, and the friction coefficient for movement of the oscillators.

The positions of the subsystems are required to satisfy the coupled constraints:

$$\left| p^i(t) - \frac{p^{i-1}(t) + p^{i+1}(t)}{2} \right| \leq 1, \quad i = 2, ..., M-1, \forall t \tag{5.14}$$

which means that each oscillator must not deviate too far from the middle of its two closest neighbors.

**Results:** Figure 5.6 shows the evolution in the first sampling interval of the normalized 2-norm error between the solution of the proposed distributed MPC method and the centralized optimum for the optimal control problem as a function of the iteration step $p$, for different values of $\alpha$. Clearly, as more iterations are performed, the error reduces. Although in [10] the recommended design parameter $\alpha$ is $\alpha_0 = \frac{s}{\rho}$, we have performed simulations with different values of $\alpha$ to show the influence of $\alpha$ on the convergence speed. We see that with the recommended $\alpha = \alpha_0$, the convergence speed is very low, and that when $\alpha$ is smaller, the algorithm converges faster. However, we cannot reduce $\alpha$ too much, there is a lower limit of $\alpha$ so that the algorithm still converges. In fact, we illustrate in Figure 5.7 that the algorithm diverges for $\alpha = 0.00001\alpha_0$.

[1] $\alpha$ is a design parameter that has to be sufficiently large. With $\alpha \geq s/\rho$ Han's method will converge [10]. For positive definite QPs we can choose $\rho$ as one half of the smallest eigenvalue of the Hessian matrix. A smaller $\alpha$ leads to a faster convergence rate, but an $\alpha$ that is too small could lead to convergence problems.

[2] The conjugate function of a function $q(\mathbf{x})$, $\mathbf{x} \in \mathbb{R}^{n_\mathbf{x}}$ is defined by: $q^*(\mathbf{y}) = \sup_{\mathbf{x} \in \mathbb{R}^{n_\mathbf{x}}} \left(\mathbf{y}^T\mathbf{x} - q(\mathbf{x})\right)$. The conjugate function $q^*$ is always convex [4].

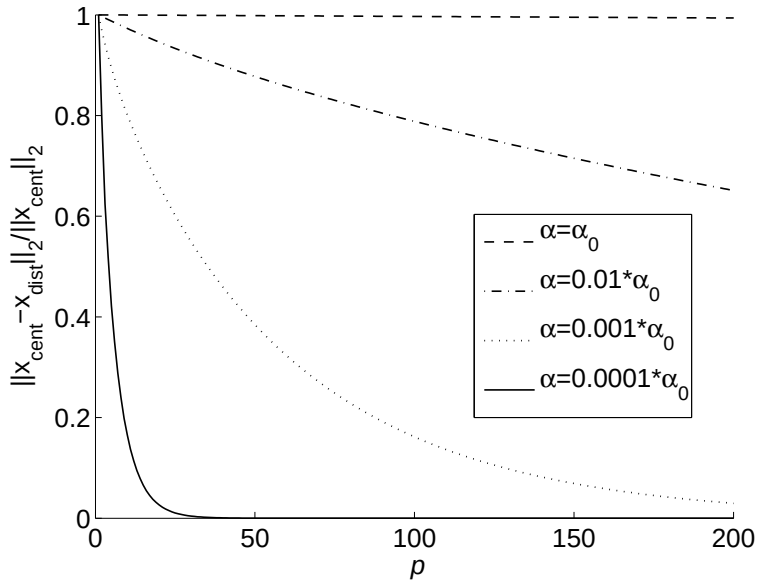



Figure 5.6: Normalized norm of difference between the centralized and the distributed solutions versus the iteration step $p$ for different values of $\alpha$.

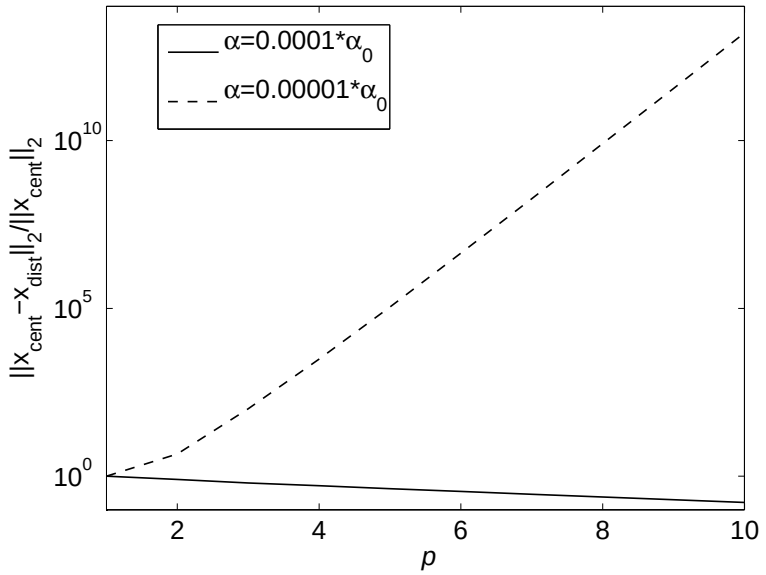



Figure 5.7: Normalized norm of difference between the centralized and the distributed solutions versus the iteration step $p$. For $\alpha = 0.00001\alpha_0$ the algorithm diverges.

# Chapter 6

# Conclusions

We have presented an overview of existing coordination methods, for hierarchical as well as distributed model predictive control (MPC) methods. Most of the existing coordination mechanisms are related to price-driven coordination. In order to achieve an overall optimum, prices are defined, e.g., by a supervisor, which are included in the objective function of each local MPC controller. Thus, the local MPC controllers take into account knowledge of the overall system. There are different ideas on how to calculate these prices, the most popular one is the dual optimization method. There, Lagrange multipliers represent the prices and are calculated as a solution of the dual problem.

Then we have assessed some of the existing coordination mechanisms. On the one hand we examined a simple distributed MPC scheme, which is based on communication only. We have verified that communication can increase overall performance of the control compared to a completely decentralized control. However, communication is not sufficient to achieve an overall optimum of the control problem. The proposed coordination scheme requires more computational power than a completely decentralized control, though it can be computed much faster than the centralized MPC.

On the other hand we have assessed price-driven coordination mechanisms based on dual optimization. This coordination mechanism leads to an overall optimal control of the distributed system, i.e., to a control performance that equals the performance of centralized MPC. However, we have seen that convergence of the method strongly depends on the parameters chosen. Unfavorable parameters can lead to slow convergence or even to divergence.

It is not always clear how the existing price-driven coordination mechanisms can be extended or improved in the future. This is a topic of ongoing research. Moreover, in a recent work [25], we have introduced a new coordination scheme. In order to achieve overall optimality, the overall objective is partly approximated by a linearization of the global objective. Hence, the coordination method requires the calculation of first order sensitivities. This method, that we refer to as gradient-based distributed dynamic optimization (GBDDO), revealed some promising properties, in particular fast convergence. Furthermore, there are no free (tuning) parameters to choose. Thus, the method is easy to implement. Hence, the related coordination mechanism is one that we will also focus on also in future research.

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