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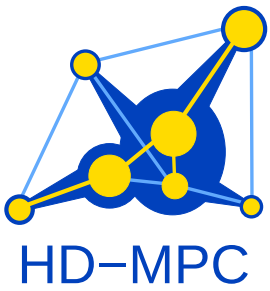

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Executive Summary

The objective of this deliverable is to provide information about the benchmark cases. Four deliverables have been proposed and are described in this deliverable:

– Four tank system,

– Chemical benchmark case,

– Electric power system,

– Heat system.

The deliverable is organized in two documents, Part I includes the description of the four tank system, and Part II (i.e., the current part) describes the other three benchmark cases.

Index

pu : Per unit (normalized quantity).

δ : Generation units load angle [rad].

ω : Generation units speed [pu].

ω_0 : Base speed (its values generally is 1 pu).

M : Rotor inertia coefficient [pu].

D : Generation units damping [pu].

Pm : Generation units mechanical power [pu].

Pe : Generation units electrical power (active power) [pu].

E'_q : Generation units quadrature axis electromotric force [pu].

E'_d : Generation units direct axis electromotric force [pu].

τ'_{d0} : Generation units direct axis transient time [s].

x_d : Generation units direct axis synchronous reactance [pu].

x'_d : Generation units direct axis transient reactance [pu].

I_d : Generation units direct axis stator current [pu].

I_q : Generation units quadrature axis stator current [pu].

E_f : Generation units field voltage [pu].

ψ^r_{qs}: Generation units quadrature axis stator flux in a rotor frame [pu].

ψ^r_{ds}: Generation units direct axis stator flux in a rotor frame [pu].

ψ^r_{0s}: Generation units zero axis stator flux in a rotor frame [pu].

ψ^r_{kq1}: Generation units quadrature axis flux of damping winding 1 in a rotor frame [pu].

ψ^r_{kq2}: Generation units quadrature axis flux of damping winding 2 in a rotor frame [pu].

ψ^r_{kd}: Generation units direct axis flux of damping windings in a rotor frame [pu].

ψ^r_{fd}: Generation units direct axis flux of field winding in a rotor frame [pu].

Y : System admittance matrix [pu].
θ : Voltage angles of generation nodes [rad].
γ : Voltage angles of load nodes [rad].
P_d : Active power demanded at load nodes [pu].
Q_d : Reactive power demanded at load nodes [pu].

Chapter 1

Chemical Benchmark Case

1.1 Introduction

Economic forces are clearly requiring chemical processes to have greater material and energy integration, which leads to increased interaction among individual process units [Luyben and Luyben, 1995]. Nowadays the required low production costs leads to the implementation of optimal control strategies. These optimal control strategies need to know almost all the issues of the process in order to find some control policy that integrate them. An interesting problem from the point of view of large, complex and networked systems is the set of three chemical reactors with three binary distillation columns and recycles streams. This document gives a detailed description of the process in order to use it as a benchmark case to develop optimal control structures.

1.2 Description of the System

The system proposed in this document is a chemical plant of generic compounds A, B, C, D, E and F. The aim is to transform the raw material A and E into the final product F at the lowest operational cost. As Figure 1.1 shows, there are two fresh feed streams A and E. There are also two product streams C and F where the main product is F.

1.2.1 The Process

The complete chemical process is composed by three chemical reactors type CSTR (Continuous Stirred Tank Reactor) that are called R1, R2 and R3, three non reactive binary distillation columns called C1, C2 and C3 and two recycle streams called RC1 and RC3. Figure 1.1 gives a Process Flow Diagram (PFD) that helps to understand the following detailed description. It must be clear for the reader that this benchmark case presents only dynamics related with the material balance, the energy and momentum dynamics are not considered as relevant issues here.

A fresh stream of A and the recycle streams RC1 and RC3 are fed to R1 where the reaction represented by Eq. (1.1) with a kinetic constant k_1 is carried out. This step produces a main intermediate product D and a byproduct C. The effluent from R1, assumed as an ideal mixture, is fed to C1 where it is separated in: top products rich in A and B called RC1, and bottom products rich in C and D. At this point the stream RC1 is fed again to R1 and the bottom stream is fed to C2. In C2, the byproduct C and the intermediate process D are separated. The top products with high concentration of C are byproducts that are removed from the process. The bottom products with high concentration of D are

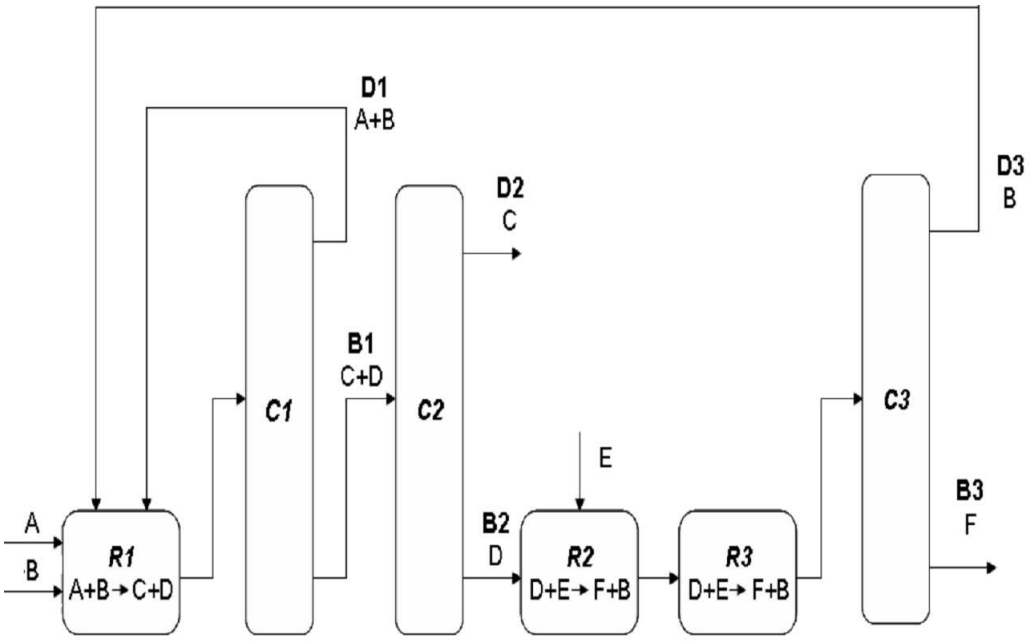

Figure 1.1: Conceptual flow sheet of the process [Scattolini, 2009]

	V (lb-mol)
R1	1000
R2	850
R3	282

Table 1.1: Reactors specifications

fed to R2. In R2 there is a fresh stream of E fed along with the bottom effluent from C2 and a new chemical reaction represented by Eq. (1.2) is carried out with a kinetic constant k_2. The products and the material without reacting from R2 pass to R3 to reach a higher conversion. Although the reaction in R2 is the same of R3, in R3 the kinetic constant is k_3 that may be different of k_2 due to effects of temperature and agitation. Then the effluent from R3 is fed to C3. In C3 the effluent from R3 is separated in bottom products with high concentration of F which is the interesting product, and top products with high concentration of B. This high B concentration stream is called RC3 and is fed again to R1. A physical characterization of each equipment is shown in Tables 1.1, 1.2, 1.3 and 1.4.

$$A + B \rightarrow C + D \tag{1.1}$$

$$D + E \rightarrow F + B \tag{1.2}$$

The physicochemical properties of the compounds and reactions are summarized in Table 1.5. The different mixtures are considered as ideal, there are not reactions inside the pipes, the transportation time in pipes is not considered and all the chemical reactions are considered as elemental.

The process variables considered in this benchmark case will be discriminated in the following groups:

C1	#	hold-up (lb-mol)
Condenser	1	33.6
Enriching Trays	8	1.36
Feed Tray	1	1.36
Stripping Trays	8	1.63
Reboiler	1	53.8

Table 1.2: Distillation column C1 specifications

C2	#	hold-up (lb-mol)
Condenser	1	32.1
Enriching Tray	8	1.27
Feed Tray	1	1.27
Stripping Trays	8	1.39
Reboiler	1	100

Table 1.3: Distillation column C2 specifications

C3	#	hold-up (lb-mol)
Condenser	1	100
Enriching Tray	8	0.837
Feed Tray	1	1.04
Stripping Trays	8	1.04
Reboiler	1	31.7

Table 1.4: Distillation column C3 specifications

	Relative volatility	k_1 (h^{-1})	k_2 (h^{-1})	k_3 (h^{-1})
A	8	10	50	141
B	6			
C	4			
D	2			
E	1.2			
F	1			

Table 1.5: Physicochemical properties of compounds and reactions

- *Controlled Variables (CV)*: There is a basic group of variables that could be used as control variables, in order to build the large amount of control strategies that could be proposed. This basic group consist of the variables depicted in Table 1.6. It is important to clarify that the maximum values were supposed as the 20% up to the nominal value due to lack of information from the real process.

Key name	Symbol	Description	Units	Nominal value	Min Max
CV1	C_{A_R1}	Concentration of A in R1	molar fraction	0.2589	0 1
CV2	C_{B_R1}	Concentration of B in R1	molar fraction	0.0356	0 1
CV3	V_{R1}	Amount of material in R1	lb-mol	1000	0 1200
CV4	$C_{i\_top\_C1}$	Concentration of A or B in the top of C1	molar fraction	A=0.9425 B=0.0468	0 1
CV5	V_{C_C1}	Amount of material in the condenser of C1	lb-mol	33.6	0 41
CV6	$C_{i\_bott\_C1}$	Concentration of C or D in the bottom of C1	molar fraction	C=0.0107 D=0	0 1
CV7	V_{R_C1}	Amount of material in the reboiler of C1	lb-mol	53.8	0 65
CV8	$C_{C\_top\_C2}$	Concentration of C in the top of C2	molar fraction	0.8274	0 1
CV9	V_{C_C2}	Amount of material in the condenser of C2	lb-mol	32.1	0 39
CV10	$C_{D\_bott\_C2}$	Concentration of C in the bottom of C2	molar fraction	0.9406	0 1
CV11	V_{R_C2}	Amount of material in the reboiler of C2	lb-mol	100	0 120
CV12	C_{D_R2}	Concentration of D in R2	molar fraction	0.148	0 1
CV13	C_{E_R2}	Concentration of E in R2	molar fraction	0.0152	0 1
CV14	V_{R2}	Amount of material in R2	lb-mol	850	0 1020
CV15	V_{R3}	Amount of material in R3	lb-mol	282	0 338

CV16	$C_{B\_top\_C3}$	Concentration of B in the top of C3	molar fraction	0.7046	0 1
CV17	V_{C_C3}	Amount of material in the condenser of C3	lb-mol	100	0 120
CV18	$C_{F\_bott\_C3}$	Concentration of F in the bottom of C3	molar fraction	0.9915	0 1
CV19	V_{R_C3}	Amount of material in the reboiler of C3	lb-mol	31.7	0 39

Table 1.6: Controlled variables

- *Manipulated Variables (MV):* This process has 18 manipulated variables depicted in Table 1.7. Again the values of the maximum were stated at the 20% up to the nominal due to the lack of information about the real process.
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Economical Aspects

Cost Function In order to define a proper cost function for this plant, each one of the associated cost will be mentioned. As the plant is divided in six stages, each stage will have some associated operational costs in the following way.

1. Stage R1: Reaction of A + B to produce C + D, by the stoichiometry becomes clear the mole conservation in this reaction. The associated costs are:
   - cost of feeding one fresh lb-mol of A in the stage R1 (cost of A + pumping)
   - cost of feeding one fresh lb-mol of B in the stage R1 (cost of B + pumping)
   - cost of feeding one lb-mol of B recycled in the stage R1 (pumping)
   - cost of converting A + B into C + D (energy requirements)
2. Stage C1: Separation of A, B, C and D, in this stage is assumed that chemical reaction is not carried out and just physical separation is achieved, the associated costs are:
   - cost of feeding one lb-mol of A + B + C + D in the stage C1 (pumping)
   - cost of separating A + B into C + D (energy requirements)
3. Stage C2: Separation of C + D into C and D, the associated costs are:
   - cost of feeding one lb-mol of C + D in the stage C2 (pumping)

Key name	Symbol	Description	Units	Nominal value	Min Max
MV1	F_{A_R1}	Flow of fresh A to R1	lb-mol/hr	100	0 120
MV2	F_{B_R1}	Flow of fresh B to R1	lb-mol/hr	0.5	0 60
MV3	F_{ef_R1}	Discharge flow from R1	lb-mol/hr	321	321 386
MV4	F_{bott_C1}	Bottom flow from C1	lb-mol/hr	241	0 290
MV5	F_{IV_C1}	Internal Vapor flow in C1	lb-mol/hr	410	0 492
MV6	F_{IR_C1}	Internal reflux in C1	lb-mol/hr	330	0 396
MV7	F_{D_C1}	Distillate flow from C1	lb-mol/hr	80	0 96
MV8	F_{bott_C2}	Bottom flow from C2	lb-mol/hr	141.5	0 171
MV9	F_{IV_C1}	Internal Vapor flow in C2	lb-mol/hr	385	0 462
MV10	F_{IR_C2}	Internal reflux in C2	lb-mol/hr	283	0 340
MV11	F_{D_C2}	Distillate flow from C2	lb-mol/hr	99.5	0 120
MV12	F_{E_R2}	Flow of fresh E to R2	lb-mol/hr	99.5	0 120
MV13	F_{ef_R2}	Discharge flow from R2	lb-mol/hr	241	0 290
MV14	F_{ef_R3}	Discharge flow from R3	lb-mol/hr	241	0 290
MV15	F_{bott_C2}	Bottom flow from C3	lb-mol/hr	100	0 120
MV16	F_{IV_C1}	Internal Vapor flow in C3	lb-mol/hr	282	0 339
MV17	F_{IR_C3}	Internal reflux in C3	lb-mol/hr	141	0 170
MV18	F_{D_C3}	Distillate flow from C3	lb-mol/hr	141	0 170

Table 1.7: Manipulated variables

- cost of separating C + D into C and D (energy requirements)
- *possible cost of disposal of C*

4. Stage R2: Reaction of D + E to produce F + B, by the stoichiometry becomes clear the mole conservation in this reaction. The associated costs are:

   - cost of feeding one lb-mol of D in the stage R2 (pumping)
   - cost of feeding one fresh lb-mol of E in the stage R2 (cost of E + pumping)
   - cost of converting D +E into F + B (energy requirements)

5. Stage R3: Reaction of D + E to produce F + B, by the stoichiometry becomes clear the mole conservation in this reaction. The associated costs are:

   - cost of feeding one lb-mol of D + E in the stage R3 (pumping)
   - cost of feeding one lb-mol of F + B in the stage R3 (pumping)
   - cost of converting D +E into F + B (energy requirements)

6. Stage C3: Separation of F + B into F and B, the associated costs are:

   - cost of feeding one lb-mol of F + B in the stage C3 (pumping)
   - cost of separating F + B into F and B (energy requirements)

All these costs must be expressed in the correct units, the units selected in this case were $[€/lb-mol_i]$ where i refers to any component. Eq. (1.3) show the units required in the function. Although the final expression is expressed in € per hours this is not relevant due to one pretend to minimize the total cost what is the same to minimize the rate of "generation of costs", due to the linearity of the cost function.

$$\left[\frac{€}{lb-mol_i}\right] * \left[\frac{lb-mol_i}{h}\right] = \left[\frac{€}{h}\right] \tag{1.3}$$

Cost Determination: The costs mentioned above will be stated in this section. They are based on the quantity of energy required in one reactor due its size, the relative volatility between the two components to be separated, the degree of separation, the value of the kinetic constant for a specific reaction in a specific reactor and finally due to the lack of components properties the pumping cost is the same for all components. The value of each cost specified above is defined in Table 1.8.

The associated costs are multiplied by the respective stream, generating the function to be minimized, Eq. (1.4) shows the final form of the cost function.

$$\begin{aligned} F = \quad & C_A^f * F_A + C_B^f * F_{B\_R1} + C_p * (D1 + D3) + \\ & C_r^{AB} * (F_A + D3 * x_A^{D3} - B1 * x_A^{B1}) + \\ & C_p * (B1 + D1) + C_s^{ABCD} * B1 * (x_C^{B1} + x_D^{B1}) + \\ & C_p * B1 + C_s^{CD} * B2 * x_D^{B2} + C_d^C * D2 + \\ & C_p * B2 + C_E^f * F_E + C_r^{DE2} * [B2 * x_D^{B2} - (B2 + F_E) * x_D^{R2}] + \\ & C_p * (B2 + F_E) + C_r^{DE3} * (B2 + F_E) * (x_D^{R2} - x_D^{R3}) + \\ & C_p * (B2 + F_E) + C_s^{BF} * B3 * x_F^{B3} \end{aligned} \tag{1.4}$$

ITEM	COST	UNIT	
Fresh A (C_A^f)	0.5	$\left[\frac{€}{lb-mol_A}\right]$	
Fresh B (C_B^f)	0.8	$\left[\frac{€}{lb-mol_B}\right]$	
Fresh E (C_E^f)	0.8	$\left[\frac{€}{lb-mol_E}\right]$	This seems to be the key raw material in the process
Pumping (C_p)	$2.2e^{-5}*\Delta h_{pump}*C_{KWh}$ (*)	$\left[\frac{€}{lb-mol_i}\right]$	Pumping requires the same energy for any component, and here was assumed that the molecular weight of each stream is roughly the same that water
Disposal of C (C_d^C)	0.2	$\left[\frac{€}{lb-mol}\right]$	
Converting A+B→C+D (C_r^{AB})	0.5	$\left[\frac{€}{lb-mol_A}\right]$	
Converting D+E→F+B (C_r^{DE2})	0.6	$\left[\frac{€}{lb-mol_D}\right]$	In R2 the energy requirements are higher, due to it is bigger than R3
Converting D+E→F+B (C_r^{DE3})	0.3	$\left[\frac{€}{lb-mol_D}\right]$	
Separating A+B+C+D into A+B and C+D (C_s^{ABCD})	$0.6*\frac{\left(x_i^D-x_i^{in}\right)}{\left(1-x_i^{in}\right)}$ (**)	$\left[\frac{€}{lb-mol_{ABCD}}\right]$	The cost of processing by mole of A+B+C+D fed, main components
Separating C+D into C and D (C_s^{CD})	$0.4*\frac{\left(x_i^D-x_i^{in}\right)}{\left(1-x_i^{in}\right)}$ (**)	$\left[\frac{€}{lb-mol_{CD}}\right]$	The cost of processing by mole of C+D fed, main components
Separating F+B into F and B (C_s^{FB})	$0.2*\frac{\left(x_i^D-x_i^{in}\right)}{\left(1-x_i^{in}\right)}$ (**)	$\left[\frac{€}{lb-mol_{BF}}\right]$	The cost of processing by mole of B+F fed, main components

(*) Where Δh_{pump} is the required change in height by pumping in meters, and C_{KWh} is the cost of KWh in €/KWh

(**) The cost depends on the degree of separation, where x_i^D, x_i^{in} are respectively the molar fraction of i in the distillate stream and the molar fraction of i in the feed stream

Table 1.8: Operational costs

Where F_i is the fresh molar flow of i in $lbmol/h$, x_j^i is the molar fraction of the component j in the stream or stage i. The stages and streams were defined in Figure 1.1 and the costs were specified in Table 1.8. Eq. (1.4) can be simplified but this expression is advantageous due to the row 1 and 2 describe the first stage and the other rows describe the subsequent stages.

1.2.2 Instrumentation and Control System

Initially, the solution for the process control problem is an open topic in this benchmark case. There is a basic set of final control elements shown in Figure 1.2. All of them are valves corresponding to the respective manipulated variable in Table 1.7. In [Luyben and Luyben, 1995] there are some proposals of control structures based on different control loops.

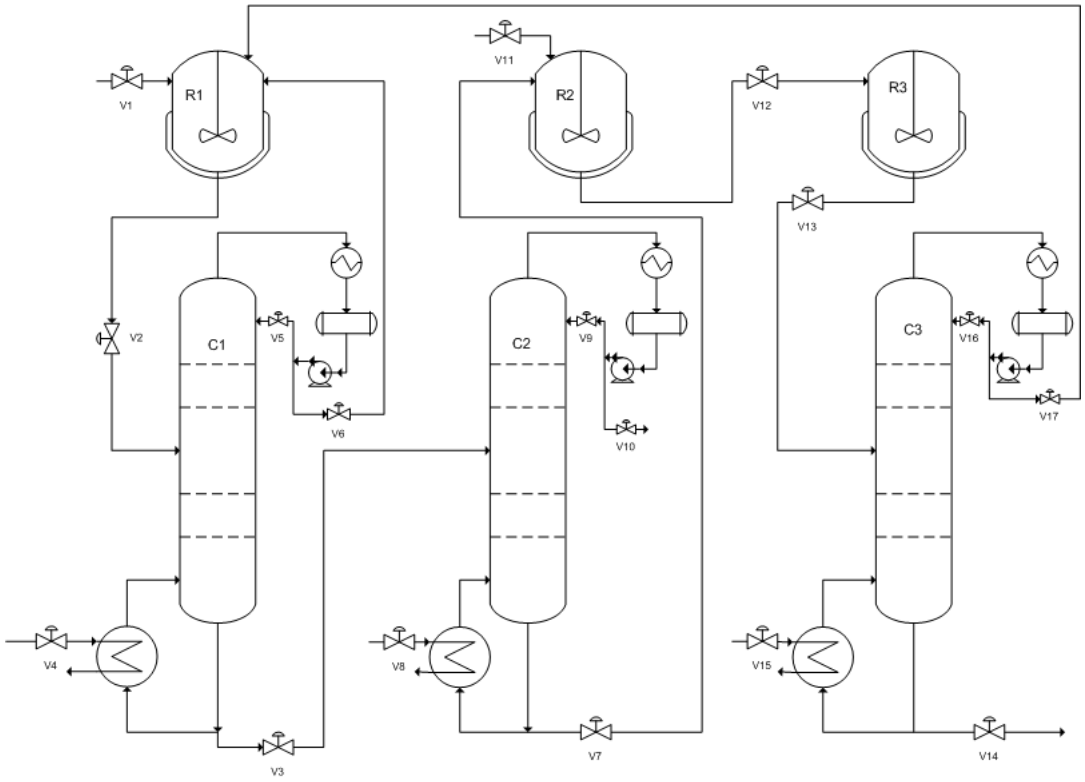

Figure 1.2: Benchmark case P& ID

The set of measurements variables are listed in Table 1.9. In order to develop a different proposals for control structures, the users can suggest additional measured, controlled or manipulated variables that can be required in some specific structure.

1.2.3 Security

Security issues are included in the limits proposed for each variable. It must be clear that the experiments will be done in simulations, the real plant is not available, and so the security levels are stated just as limits in the variables.

Key Name	Name	Description
MeV1	L_{R1}	Level on R1
MeV2	C_{B_R1}	Concentration of B in R1
MeV3	F_{ef_R1}	Discharge flow from R1
MeV4	L_{R_C1}	Level in reboiler of C1
MeV5	F_{IR_C1}	Flow of internal reflux in C1
MeV6	L_{C_C1}	Level of the condenser in C1
MeV7	$C_{i\_bott\_C1}$	Concentration of C or D in bottom products of C1
MeV8	F_{IR_C2}	Flow of internal reflux in C2
MeV9	L_{C_C2}	Level of the condenser in C2
MeV10	$C_{C\_top\_C2}$	Concentration of C in top products of C2
MeV11	L_{R_C2}	Level in reboiler of C2
MeV12	F_{bott_C2}	Bottom flow from C2
MeV13	C_{E_R2}	Concentration of E in R2
MeV14	L_{R2}	Level on R2
MeV15	L_{R3}	Level on R3
MeV16	L_{R_C3}	Level in reboiler of C3
MeV17	$C_{F\_R\_C3}$	Concentration of F in reboiler of C3
MeV18	F_{IR_C3}	Flow of internal reflux in C3
MeV19	L_{C_C3}	Level in condenser of C3

Table 1.9: Measured variables

1.2.4 Process Models

The proposed model for this benchmark case is based on first principles, the basic material balances lead the equations describing the dynamic behavior. This model was proposed by Scattolini [Scattolini, 2009].

Dynamic model of the reactors

Consider a chemical reactor and assume that:

- all the energy phenomena are negligible
- the hydraulic phenomena are all at the steady state
- there is perfect mixing inside the reactor;

Define:

- q_{I_j} as the volumetric flow rate of the j-th input;
- c_{I_ji} as the concentration of the i-th component in the j-th input flow rate;
- V as the reactor volume;

- c_i as the concentration inside the reactor of the i-th component;
- q_{O_j} as the volumetric flow rate of the j-th output;
- c_{O_ji} as the concentration of the i-th component in the j-th output flow rate;
- n_i as the number of input components;
- n_0 as the number of output components;
- n_r as the number of reacting components;
- k as the reaction constant;

The mass balance of the i-th component inside the reactor is then given by Eq. (1.5).

$$\frac{dc_i}{dt} = \frac{1}{V} * \left[\sum_{j=1}^{n_i} c_{I_{ij}} . q_{I_j} - \sum_{j=1}^{n_0} c_{O_{ij}} . q_{O_j} \right] \pm k \prod_{r=1}^{n_r} x_r \tag{1.5}$$

Assuming that inside the reactor there are n components, the model will be described by a system of n differential equations besides one more equation describing the hydraulic equilibrium, that is shown by Eq.(1.6).

$$\sum_{j=1}^{n_i} q_{I_j}(t) = \sum_{j=1}^{n_0} q_{O_j}(t) \tag{1.6}$$

Finally, note that the dynamic model previously derived can be expressed in terms of molar fractions x_i, instead of concentrations c_i by defining Eq. (1.7).

$$x_i = \frac{c_i}{\sum_{j=1}^{n} c_j} \tag{1.7}$$

Then, with an obvious meaning of symbols, the dynamic equations can be written as Eq. (1.8) shows.

$$\frac{dx_i}{dt} = \frac{1}{V} \left[\sum_{j=1}^{n_i} x_{I_{ij}} . q_{I_j} - \sum_{j=1}^{n_0} x_{O_{ij}} . q_{O_j} \right] \pm k \prod_{r=1}^{n_r} x_r \tag{1.8}$$

and finally Eq. (1.9) shows the relation among molar fractions.

$$\sum_{j=1}^{n} x_j = 1 \tag{1.9}$$

Dynamic model of the distillation columns

The simplified model of the tray distillation column here considered assumes that it is composed by five sections:

1. Condenser
2. Enriching section

3. Feed tray
4. Stripping section
5. Reboiler

where the enriching and stripping sections can be composed by a variable number of trays. A schematic diagram of the column is shown in Figure 1.3, where:

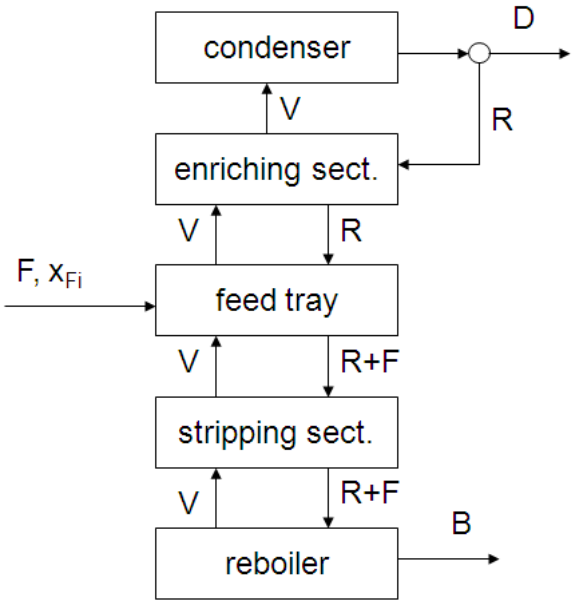

Figure 1.3: Schematic representation of a distillation column

1. V is the vapor flow rate;
2. R is the reflux flow rate;
3. D is the distillate flow rate;
4. B is the flow rate of the bottom product;
5. F is the feed flow rate;
6. x_{fi} is the liquid molar fraction of the i-th component into the feed flow rate.

Assume that the mixture is formed by N components and let

- x_i is the liquid molar fraction of the i-th component (i = 1,2,...,N);
- y_i is the vapor molar fraction of the i-th component (i = 1,2,...,N);
- α_i volatility of the i-th component (i = 1,2,...,N);
- α_{ij} relative volatility of the i-th component with respect to the j-th component (i; j = 1,2,...,N).

Straightforward computations allow to conclude that the relation among the liquid and the vapor molar fractions is given by the set of linear equations shown in Eq. (1.10).

$$\begin{bmatrix} 1+\alpha_{1N}\frac{x_1}{x_N} & \alpha_{1N}\frac{x_1}{x_N} & \dots & \alpha_{1N}\frac{x_1}{x_N} \\ \alpha_{2N}\frac{x_2}{x_N} & 1+\alpha_{2N}\frac{x_2}{x_N} & \dots & \alpha_{2N}\frac{x_2}{x_N} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \alpha_{(N-1)N}\frac{x_{N-1}}{x_N} & \alpha_{(N-1)N}\frac{x_{N-1}}{x_N} & \dots & 1+\alpha_{(N-1)N}\frac{x_{N-1}}{x_N} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_{N-1} \end{bmatrix} = \begin{bmatrix} \alpha_{1N}\frac{x_1}{x_N} \\ \alpha_{2N}\frac{x_2}{x_N} \\ \cdot \\ \cdot \\ \cdot \\ \alpha_{(N-1)N}\frac{x_{N-1}}{x_N} \end{bmatrix} \tag{1.10}$$

The mathematical model of the column is derived under the fundamental assumption that the energetic phenomena are negligible, so that only mass balance equations are used. Moreover, the following simplifying hypothesis are introduced.

- The pressure inside the column is constant
- The vapor flow rate V can be directly manipulated (the reboiler has no dynamics)
- The liquid (R) and vapor (V) flow rates are constant inside the column
- The hydraulic dynamics is negligible with respect to the dynamics of the concentrations
- The vapor hold-up on the trays is negligible with respect to the liquid hold-up;
- The Murphee efficiency is constant for any (i-th) component and any (j-th) tray, as Eq. (1.11) and Eq. (1.12) show (the user can modify the efficiency in order to make more real the process).

$$E_{j_i} = \frac{y_{j_i} - y_{(j-1)_i}}{y^*_{j_i} - y_{(j-1)_i}} = 1 \tag{1.11}$$

$$y^*_{j_i} = \frac{\alpha_{iN}.x_i}{1+(\alpha_{iN}-1)x_i} \tag{1.12}$$

Define the following quantities:

- H_j as the liquid hold-up in the j-th tray;
- H_a as the liquid hold-up in the feed tray;
- N_a as the number of trays in the enriching section;
- N_e as the number of trays in the stripping section;
- x_{ji} as the liquid molar fraction of the i-th component in the j-th tray;
- y_{ji} as the vapor molar fraction of the i-th component in the j-th tray;
- x_{ai} as the liquid molar fraction of the i-th component in the feed tray;;
- x_{ti} as the liquid molar fraction of the i-th component in the top product;
- x_{bi} as the liquid molar fraction of the i-th component in the bottom product;
- F as the feed flow rate;

- x_{f_i} as the liquid molar fraction of the i-th component in the feed flow rate;
- α_{iN} as the relative volatility of the i-th component with respect to the N-th component.

Denoting by the index $j = 1$ the reboiler and by the index $j = N_p$ the condenser and defining:

- $x_{1i} = x_{bi}$
- $x_{Npi} = x_{ti}$
- $N_p = N_a + N_e + 3$ as the total number of trays, including reboiler and condenser

the mass balance for any tray and for any i-th component is:

1. Static balance of the flow rates at the condenser:

$$V = R + D \tag{1.13}$$

2. Static balance of the flow rates at the reboiler:

$$V + B = R + F \tag{1.14}$$

3. Dynamic balance at the reboiler (the dot above the x means dx/dt):

$$H_{1i}\dot{x}_{1i} = -Vy_{1i} + (R+F)x_{2i} - Bx_{1i} \tag{1.15}$$

4. Dynamic balance in the stripping section ($j = 2, ..., N_e + 1$):

$$H_{ji}\dot{x}_{ji} = (R+F)(x_{(j+1)i} - x_{ji}) + V(y_{(j-1)i} - y_{ji}) \tag{1.16}$$

5. Dynamic balance at the feed tray:

$$H_a\dot{x}_{ai} = Rx_{(Ne+3)i} - (R+F)x_{ai} + V(y_{(Ne+1)i} - y_{ai}) + Fx_{f_i} \tag{1.17}$$

6. Dynamic balance in the enriching section ($j = N_e + 3, ..., N_p - 1$):

$$H_j\dot{x}_{ji} = R(x_{(j+1)i} - x_{ji}) + V(y_{(j-1)i} - y_{ji}) \tag{1.18}$$

7. Dynamic balance at the condenser:

$$H_{Np}\dot{x}_{Npi} = Vy_{(Np-1)i} - (R+D)x_{Npi} \tag{1.19}$$

The complete model of a distillation column with N_p trays is then described by Eq. (1.13) to Eq. (1.19) , written for any component, besides the additional Eq. (1.10), (1.11) and (1.12).

1.2.5 General Operating Conditions

The general operating conditions were described in Tables 1.6, and 1.7. The values of other variables become as natural result of simulation experiments.

1.2.6 Experiment Implementation

Matlab/Simulink

The experiment implementations are subject to the availability of one version 7.0.1 of Matlab/Simulink ®, a simulator of the chemical benchmark example can be downloaded at the internet address:

$$ftp://ftp.elet.polimi.it/users/Riccardo.Scattolini/$$

In order to use the simulator, proceed as follows:

- To run the Matlab file ChemBenchData.m which initializes all the plant parameters and all the constant inputs. It is possible to edit this file and change anyone of these parameters.
- To run the Simulink file ChemBench.mdl; note that in the simulator there is the block ”plots” where all the variables are plotted and stored.

OpenModelica

A simulator version in OpenModelica 1.4.5 is also available. To use this version proceed as follow:

- There must be the follow five files:
  1. Distillation_Column.mo
  2. Reactor_CSTR.mo
  3. Plant_inputs_outputs.mo
  4. ZOG.mo
  5. Chemical_Plant.mo
- These files were created using a version of OpenModelica for the sake of accessibility, but other programs as Modelica or Dymola can be used to run this model.
- First it is necessary to load the files mentioned above. This can be made using the command load(”specific file path”) in the OMShell of OpenModelica, e.g.

  load(”C:/Modelica_Chemical_Plant/Reactor_CSTR.mo/”)

- Then, a simulation routine must be called by the command simulate, just the file Chemical_Plant.mo must be called in this routine in the follow way:

  simulate(Chemical_Plant, startTime=t_0, stopTime=t_end, numberOfIntervals=500, tolerance=1e-6).

1.3 Conclusions

This document gives a complete description of one process with remarkable importance from the point of view of control of large, complex and networked systems. The control structures are an open issue, so the performance indexes are not defined yet. The reader can propose a proper structure and define its performance index in a reasonable way. The main goal of this document was to present a complete description of an interesting process, in order to provide the necessary tools to develop the appropriate control structure.

Chapter 2

Electric Power System Benchmark

2.1 Introduction

In the open literature about electric power systems, it is commonly found systems as test beds to prove new control systems. However, model parameters and operational conditions of these systems are not available, hindering the research process. It mainly causes lost of objectivity in the evaluation of proposed control schemes, increase resources and time spent to develop validation models, and difficult the performance comparison of different control schemes.

To overcome the troubles listed before, it is presented a whole account about electric power systems, in particular New England power system case.

2.2 Description of the System

New England electric power system is composed by 10-machines 39-buses, interconnected among them by transmission lines as it is shown in Figure 2.1.

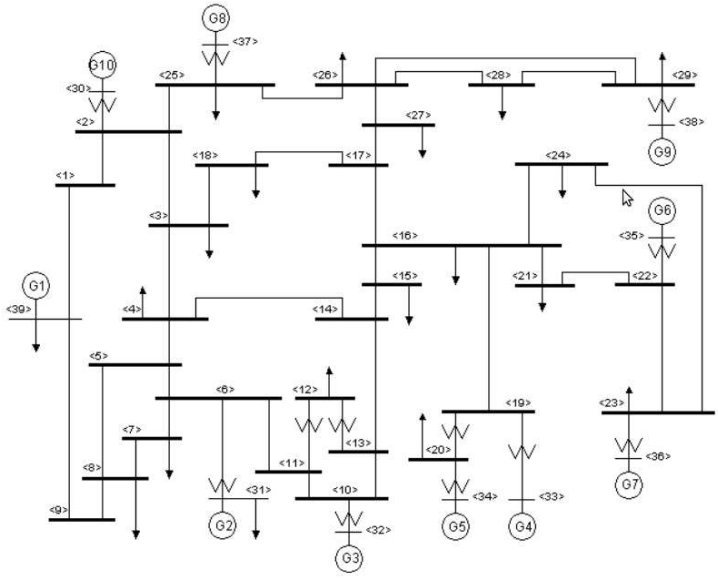

Figure 2.1: 10-machine 39-bus IEEE benchmark system

This power system has been widely used in open literature as a study case mainly to prove new optimal power dispatch schemes, or new decentralized control schemes. In the cases of optimal power dispatch, the main objective is to minimize active and reactive power losses in transmission lines while quality of service is guaranteed (voltage magnitude and frequency inside the region bounded by the regulation constraints). In the cases of control schemes, New England power system has been used to demonstrate the performance of control schemes based on local information in networked systems.

The processes associated with the system shown in Figure 2.1 are generation and transmission of electric power to customers. The generation process is carried out in power plants. In these places the potential energy of the water or the expansion of superheated steam or gas in a turbine is converted in rotational energy to generate electric energy. Electric energy is transported from the power plants to the customers or load centers, across of different voltage levels transmission networks (transmission process). In the case of power system shown in Figure 2.1, two voltage levels are considered: $15KV$ at terminals of generation units and $400KV$ in the transmission network. All loads in the transmission network are fed at $400KV$. Each generation unit feed a local load with the following features:

1. Active power demand: $2MW$
2. Reactive power demand: $1MVAR$
3. Feed voltage: $15KV$

The main objective of an electric power system is to feed the customers power demand keeping the voltage profile across the transmission network as good as possible, and the frequency in the same value. This means, to maintain voltage and frequency magnitude at different network buses between the limits allowed for the regulatory legislation. Commonly voltage must be inside 0.95 to 1.05 $V\ pu$, and frequency inside 0.99 to 1.01 $Hz\ pu$. To make it possible, generation unit's voltages and speeds are controlled.

2.2.1 Controlled Variables

Electric power systems have a hierarchical control structure. This control structure is necessary because there are many facts that difficult implementations of other control structures. Some of them are: system dimension; information uncertainties due to system granularity (several different elements interacting among them); different time-scales responses; interaction between continuous and discrete dynamic systems.

Commonly, control of electric power systems has two hierarchies:

1. Primary control
2. Secondary control

Primary control hierarchy is composed by the system devices controls. It is entrust of regulating devices behaviors and power oscillations in the system. In power system shown in Figure 2.1, only generation units controllers are considered. These control systems are: voltage control, speed regulator and power system stabilizer.

Voltage control consists in regulate generation unit field voltage to maintain in its terminals a desired voltage. Commonly it is made using solid-state power electronic devices, like Thyristors. In [Feltes et al, 2005] all IEEE terminal voltage control schemes are presented.

Speed regulation can be made sensing the speed of the generation unit or from the voltage wave frequency. Speed regulators has three main elements, it does not matter if the power plant is hydraulic or thermal. These elements are:

1. Statism: the ratio between changes in frequency and changes in mechanical power. Its common values are: 1, 0.01, 0.05. Statism allows adjusting active power compensation of each machine to face up frequency changes.

2. Governor: the element in charged to generate the control law for speed regulation. There are many topologies of governors. Some of them are presented and described in [Kundur, 1994].

3. Turbine: the final control element of the speed control scheme. There are several turbine models used in speed regulation. In [Kundur, 1994] some of them are presented.

Control of power oscillations is made using a power system stabilizer. This is a cascade of filters tuned in the bandwidth in which power oscillations are present. The aim of introduce this device, is to give a phase gain to terminal voltage control in the selected bandwidth. Thus power oscillations can be damped. Commonly, power system stabilizers are tuned using root-locus technique.

Secondary control hierarchy generally is centralized in the national control dispatch. This hierarchy is in charge of compute voltage and speed references for the primary control, based on network information. The steps followed to compute the primary control references are listed below.

1. From SCADA system, buses voltages, buses active and reactive power, steady of the protections and interrupters is taken.

2. With this information, a matrix of the system elements interconnection is made.

3. To solve a load flow and to estimate voltage angles.

4. With the results of previous steps a stability analysis is made.

5. If the stability results satisfy operational criteria, reference values are assigned to each generation unit.

In the case of optimal power dispatch, load flow routine is replaced by an optimal load flow routine. It does not matter which load flow routine was used, in this hierarchy generation units are modeled as voltage sources and in some cases its control schemes are taken into account; transmission lines are taken as RL series circuits and transformers are assumed as constant impedances.

2.2.2 Manipulated Variables

In the classic hierarchical control structure of power systems, typical manipulated variables are:

1. Primary control

   a) Generation units field voltage, for terminals voltage control.

   b) Gate-opening or servo-motor position, for speed regulation.

   c) Tap position of transformers and shunt compensators.

2. Secondary control

   a) Terminal voltage reference of generation units.
   b) Speed reference.
   c) Voltage reference of tap-changer transformers and shunt capacitors.
   d) Active and reactive power reference or power plants or generation units.

However, for simulation purposes only few of the variables listed before are considered, because the selection of the variables depends of the simulation aim or focus. For example, if the simulation aim is to prove a new control algorithm for generation units, only primary control variables are taken into account. But if the simulation focus is show a new power control dispatch, only secondary control manipulated variables are considered.

There are few cases in which variables of primary and secondary power systems control are considered. One of them is the simulation of new hierarchical regulation schemes (automatic generation control, secondary voltage regulation...).

2.2.3 Other Process Variables

In electric power generation and transmission, there are some variables which are not described yet. These variables are the states and the perturbations of the system.

System States

The states of the systems typically are the same of the generation unit's model, because it is assumed that the network dynamic response is too fast than the machines dynamics. However, depending on the models used to represent the transformers and transmission lines behaviors, the states of the system can be increased.

The generation units models and its states are:

1. Second order model: in this model, the states are the load angle and the speed of the generation unit. This model is given by the equation 2.1

$$\begin{aligned} \dot{\delta} &= \omega_0(\omega - 1) \\ \dot{\omega} &= \frac{1}{M}[-D(\omega - 1) + P_m - P_e] \end{aligned} \tag{2.1}$$

2. Third order model: in this model, the states are the load angle, the speed and the electromotric force in quadrature axis of the generation unit. Direct and quadrature quantities results after applies the Park's transformation to the generation units model in natural or *ABC* coordinates. Third order generators model is given by 2.2.

$$\begin{aligned} \dot{\delta} &= \omega_0(\omega - 1) \\ \dot{\omega} &= \frac{1}{M}[-D(\omega - 1) + P_m - P_e] \\ \dot{E}'_q &= \frac{1}{\tau'_{d0}}[-E_q - (x_d - x'_d)I_d + E_f] \end{aligned} \tag{2.2}$$

3. Fourth order model: in this model, the direct axis of electromotric force also is included as an state. Then the fourth order model can be written as 2.3.

$$\begin{aligned}
\dot{\delta} &= \omega_0(\omega - 1) \\
\dot{\omega} &= \frac{1}{M}[-D(\omega - 1) + P_m - P_e] \\
\dot{E}'_q &= \frac{1}{\tau'_{d0}}[-E_q - (x_d - x'_d)I_d + E_f] \\
\dot{E}'_d &= \frac{1}{\tau'_{d0}}[-E_q - (x_d - x'_d)I_q]
\end{aligned} \tag{2.3}$$

4. Eighth order model: in this model, the electrical dynamics of the machine are computed from direct, quadrature and zero axis stator fluxes behaviors. Also damping and excitation windings fluxes are considered. Thus the states are: stator fluxes, damping fluxes, excitation flux, load angle and speed. This model can be formulates as 2.4.

$$\begin{aligned}
\dot{\psi}^r_{qs} &= \omega_0(v^r_{qs} + r_s i^r_{qs} - \frac{\omega_r}{\omega_b}\psi^r_{ds}) \\
\dot{\psi}^r_{ds} &= \omega_0(v^r_{ds} + r_s i^r_{ds} + \frac{\omega_r}{\omega_b}\psi^r_{qs}) \\
\dot{\psi}^r_{0s} &= \omega_0(v^r_{0s} + r_s i^r_{0s}) \\
\dot{\psi}^{'r}_{kq1} &= \omega_0(v^{'r}_{kq1} - r_{kq1} i^{'r}_{kq1}) \\
\dot{\psi}^{'r}_{kq2} &= \omega_0(v^{'r}_{kq2} - r_{kq2} i^{'r}_{kq2}) \\
\dot{\psi}^{'r}_{kd} &= \omega_0(v^{'r}_{kd} - r_{kd} i^{'r}_{kd}) \\
\dot{\psi}^{'r}_{fd} &= \omega_0(v^{'r}_{fd} - r_{fd} i^{'r}_{fd}) \\
\dot{\delta} &= \omega_0(\omega - 1) \\
\dot{\omega} &= \frac{1}{M}[-D(\omega - 1) + P_m - P_e]
\end{aligned} \tag{2.4}$$

Assuming that network response is too fast than machines response, interconnections among generation units and loads can be formulated as 2.5 to 2.8.

$$0 = Pe_i + \sum_{\substack{j=1 \\ j\neq i}}^{n} [Y_{ij}V_iV_j\sin(\theta_i-\theta_j)] + \sum_{k=n+1}^{n+m+1} [Y_{ik}V_iV_k\sin(\theta_i-\gamma_k)] \tag{2.5}$$

$$0 = Qe_i - Y_{ii}V_i^2 - \sum_{\substack{j=1 \\ j\neq i}}^{n} [Y_{ij}V_iV_j\cos(\theta_i-\theta_j)] + \sum_{k=n+1}^{n+m+1} [Y_{ik}V_iV_k\cos(\theta_i-\gamma_k)] \tag{2.6}$$

$$0 = -Pd_k + \sum_{i=1}^{n} [Y_{ki}V_kV_i\sin(\gamma_k-\theta_i)] + \sum_{\substack{l=n+1 \\ l\neq k}}^{n+m+1} [Y_{kl}V_kV_l\sin(\gamma_k-\gamma_l)] \tag{2.7}$$

$$0 = -Qd_k - Y_{kk}V_k^2 - \sum_{i=1}^{n} [Y_{ki}V_kV_i\cos(\gamma_k-\theta_i)] + \sum_{\substack{l=n+1 \\ l\neq k}}^{n+m+1} [Y_{kl}V_kV_l\cos(\gamma_k-\gamma_l)] \tag{2.8}$$

where Y_{ij} is the magnitude of the $i-$ th element of system $Ybus$ matrix, and

$$Pe_i = \frac{E'_{qi}V_i\sin(\theta i-\delta i)}{x'_{di}} \tag{2.9}$$

$$Qe_i = \frac{V_i^2}{x'_{di}} - \frac{E'_{qi}V_i\cos(\theta i-\delta i)}{x'_{di}} \tag{2.10}$$

In all models shown before, only speed is a measured state, all of the other states must be observed.

Perturbations

Another class of variables that are not considered yet are the disturbances. In electric power systems, like in other dynamic systems, perturbations can be classified into measured and unmeasured.

Measured perturbations are, basically, changes in reference values due to changes in power dispatch, or power demand. Unmeasured perturbations are changes in loadability of the system, systems faults, and lightning discharges over lines, human errors and, in some places, variations in the network topology due to attacks against network infrastructure.

2.2.4 Dynamics Characterization

Electric power systems are composed by several elements interacting among them. Some of them have continuous dynamics, like generators, transmission lines and transformers, others have discrete dynamics, like shunt compensators, flexible AC transmission systems and load tap-changer transformers. Each one of these elements have its own time-scale response. Electric power system devices are listed in order of time scale response (fast to low) below:

1. Transmission lines: it responds immediately.
2. Automatic Voltage Regulators: it responds in order of milliseconds.
3. Flexible AC transmission systems: it responds in order of milliseconds.
4. Generator Voltage Dynamics: it responds in order of milliseconds to seconds.

5. Shunt compensators: it responds in order of seconds to tens of seconds.
6. Speed Regulators: it responds in order of tens of seconds.
7. Generator Speed Dynamics: it responds in order of tens of seconds.
8. Load Tap-Changers: it responds in order of minutes.
9. Dynamic Loads: it responds in order of tens of minutes to hours.

2.3 Instrumentation and Control Systems

After describes generation and transmission of electric power processes, the idea is to depict control systems involved in that processes. In following subsections, architectures of control systems, sensors, actuators, hardware and software used to implement primary and secondary power systems control are shown.

2.3.1 Architecture of Control Systems

Main architecture of control in electric power systems is the hierarchical architecture depicted in previous section. In each hierarchy there are different control structures, depending on controlled device, manufacturer, device power capability and voltage level.

Primary Control Architecture

Primary control architectures are those in charged to regulate the devices behavior. Primary control architecture is composed by:

1. Automatic Voltage Regulators: its main objective is to control generation units voltage. Automatic voltage regulators can be analogous or digital, depending on the age of the device. This device has three main control loops, listed below.
   a) Field current regulator: this control loop is entrust of regulate the excitation winding temperature and the reactive power supplied by the generation unit to the system.
   b) Compensation loop: this is an additional control loop. Its aim is gives an additional signal to compensate the voltage drop between terminals voltage and transmission substation.
   c) Direct loop: also called main control loop, its aim is to regulate the generation unit terminal voltage, to provide an additional damping of electromechanical oscillations, and to attenuate the effect of transient voltage phenomena in the whole system behavior.

   In [Feltes et al, 2005], IEEE automatic voltage regulators models are depicted.
2. Speed Regulator: its main objective is to regulate the rotor speed. Speed regulators are electromechanical devices. Its control structure is associated with manufacturer and class of power plant (thermal or hydraulic).
3. Power System Stabilizer: its main objective is to bring to automatic voltage regulator a phase gain inside the selected bandwidth, to damp power system oscillations. Power system stabilizers are composed by a cascade of filters and lead-lag networks. Depending on the interactions between filters and lead-lag networks, power system stabilizer can be classified in:

a) Single band power system stabilizer
b) Multiband power system stabilizer.

It does not matters the class of power system stabilizer to use, both single band and multiband power systems stabilizers are tuned using root-locus technique.

Power system stabilizers are manufactured as both: analogous and digital types. Its inputs include:

a) Speed
b) Frequency
c) Active power
d) Accelerating power
e) Integral of accelerating power

A best description of power systems stabilizers and some techniques for tuning these devices are shown in [Pac. G & E Co.].

4. Tap changers transformers control: tap changers transformers are the most used devices in electric power systems to regulate load voltages. Its main objective is to determinate the transformation ratio necessary to regulate, high or low transformer winding voltage at desired value.

   This control structure is conformed by a comparator, a death band and a selecting time. Comparator computes the deviation between measured and desired voltage value. If the deviation value is outside of death band along time interval greater than selecting time, a step change in transformer ratio occurs, if its limits are not achieved. Changes in transformer ratio are discrete dynamics.

5. Flexible AC transmission devices control: its function is to compute the necessary reactive power injection or absorption, or the susceptance value to maintain voltage bus magnitude in an specific value.

Secondary Control Architecture

Secondary control architecture are conformed by all systems in charged of regulate transmission network behavior. This control systems generates policies to guarantee that all system variables (voltages, power flows, frequency, angles), are inside regions bounded by regulation margins, security levels and operational constraints. Main control architectures belonging to secondary control of power systems are:

1. Automatic Generation Control: its function is to regulate the balance between power generation and demand (secondary frequency control).

2. Automatic Load Disconnection Scheme: its function is to disconnect loads and power plants to regulate systems frequency, when automatic generation control policies are not enough to restore the equilibrium between power generation and demand.

3. Secondary Voltage Regulation: its function is to maintain voltage profile across the power network according to regulation policies while optimizes the use of reactive power sources (generators, tap changer transformers, shunt capacitors, flexible AC transmission systems).

2.3.2 Sensors and Actuators

Sensor and actuators are important elements in power systems, like in other dynamic systems. They allow knowing the real value of some variables and manipulate some others, to regulate the whole systems behavior. In power systems, sensors and actuators belongs to primary or secondary control architectures. They are listed and briefly depicted below.

Primary Control Architectures

In real power systems, there are several sensors and actuators bringing necessary information and allowing manipulate some variables to regulate power plants and network devices behavior. However, in simulations only few of them are taken into account for control purposes. Sensors and actuators used in simulation control structures belonging to primary control architectures are:

1. Sensors
   a) Taco-generators or Frequency-meters: it allows to estimate rotor speed of generation units.
   b) Potential and Current Transformers: it allows to reduce voltage and current magnitudes. These signals are used to compute power flows magnitude and direction, and voltage magnitudes.
   c) Rectifiers: after amplitude reduction, rectifiers allow to take signals comparable with its desired values to regulate system devices behavior.
2. Actuators
   a) Inverters: they are used to manipulate field voltage in generation units, and to compute power injection or absorption in compensation systems.
   b) Proportional Valves and/or Servo-mechanisms: they are used to modify gate opening or water injection (depending on type of turbine and type of power plant), with the purpose of regulate rotor speed.
   c) Turbines: they are used to convert potential energy of water, or water superheated steam, in kinetic energy, and transmit it to generator unit's rotors.
   d) Motors: are used to change transformer ratio in tap changer transformers.

Secondary Control Architectures

In secondary control architectures the main element to sense variables is the remote terminal unit. Remote terminal unit transmit voltage magnitudes, frequency and power flows to SCADA system of national dispatch center. With this information, secondary control architectures depicted before computes its own control policies.

The computed policies are applied to power system using:

1. Relays: in the case of connection and disconnection of power system elements.
2. Primary controls: in the case of changes in power system reference values.

In simulations, sometimes voltage angles are assumed known, but in real systems this variables are not measured, then this variables must be estimated or observed.

2.3.3 Monitoring and Control: Hardware and Software

Monitoring of electric power systems is made through SCADA systems. Basically, there are two SCADA systems in power networks monitoring:

1. Power plant SCADA's: in which all power plant variables and devices are supervised.
2. Whole system SCADA: in which bus voltage magnitudes, frequency and power flows are displayed and supervised.

2.4 Security

Security is an important issue in electric power systems because at each time high amounts of energy flows across the power networks. This fact makes each device in power systems has its own protection scheme. These protection schemes are tuned to react in cooperation with other network protection elements to avoid failures propagation, or undesirable behaviors due to network contamination or bad operation of any other device. Main protection elements in power networks are:

1. Relays: its main functions are to isolate system failures by disconnection of bad operating devices, and to guard power network devices when they are working in non-desirable conditions (overload, harmonic environments, reheating conditions).
2. Surge Arrester: its main function is to guard network devices when there are over-voltages in its feeder terminals.
3. Shielding: its main functions is to guard network devices and phase wires from lightning.

2.5 Process Models

The 10-machines 39-buses electric power system showed in Figure 2.1 is composed basically by three different subsystems:

1. Generation Units: this model covers the generator, the automatic voltage and the speed regulator model.
2. Transformers
3. Transmission lines

For simulation purposes, any protection scheme, any secondary control system and any compensation systems as tap changer transformers, shunt capacitors or flexible AC transmission systems was considered.

The dynamic models of each subsystem suggested to implement the benchmark system are presented in the following subsections.

2.5.1 Generation Units

The generation units model used to represent the machine's behavior, was the eight-order *dq* synchronous machine model, yield by applying Park's transformation to the natural coordinates synchronous machine model. This model takes into account the dynamics of the stator, the field and damper windings. The equivalent circuit model is presented in the rotor reference frame in figure 2.2. All rotor parameters and electrical quantities are viewed from the stator (they are identified by the primed variables).

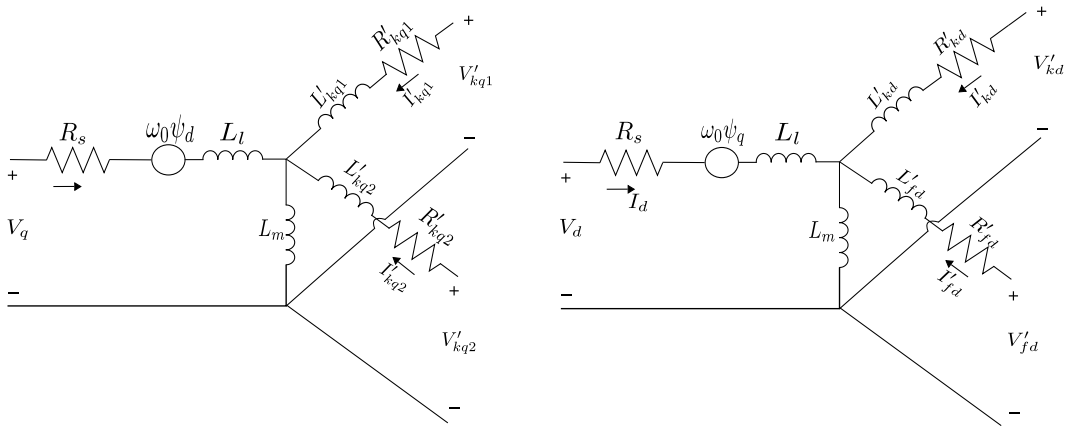

Figure 2.2: Equivalent circuit of three-phase synchronous machine

In figure 2.2:

- *d* and *q* means direct and quadrature axis quantities.
- *R* and *S* represent rotor and stator quantities.
- *l* and *m* are leakage and mutual inductances.

- f and k are the field and damper winding quantities.
- the prime, $(')$, means that quantities are referenced and the super-index r entail that reference frame is the machine's rotor.

The model of the generation units is depicted by the equation 2.4, and it was selected because this is the closest model to real machine's behavior. To take a benchmark system close to real power plants, voltage and speed regulators, and power system stabilizer also are considered. In this system, manipulated variables are field voltage and mechanical power. Controlled variables are terminals voltage and speed. The block diagrams of the control systems implemented are displayed in figure 2.3.

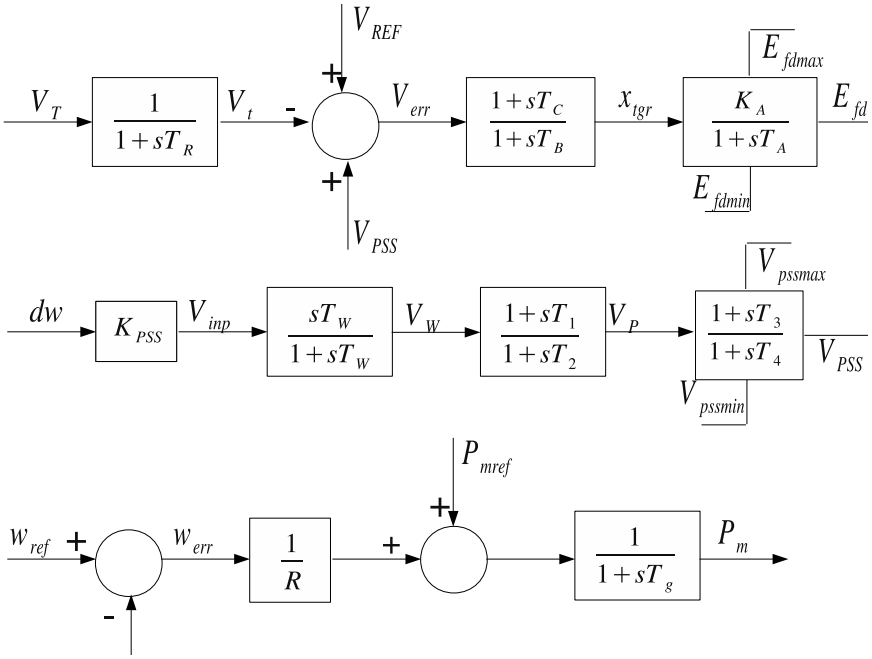

Figure 2.3: Control systems: Automatic voltage regulator (top), Power system stabilizer (middle), Speed regulator (below)

2.5.2 Transformers

To coupling the generation units with the transmission network, generation units are connected to a step-up transformer. This coupling allows reduce losses in transmission lines increasing voltage levels in power network. In the benchmark system depicted in this work, transformers increase generation voltages from $15KV$ to $400KV$. To model the transformers, its equivalent mono-phase circuit presented in the figure 2.4 was used.

Dynamic model derived from equivalent circuit displayed in Figure 2.4 for each pair of phases of tri-phase power system are given by equation 2.11 and 2.12.

$$\dot{\psi}_1 = \omega_b(v_1 - r_1 i_1) \tag{2.11}$$

$$\dot{\psi}_3 = \omega_b(v_3 - r_3 i_3) \tag{2.12}$$

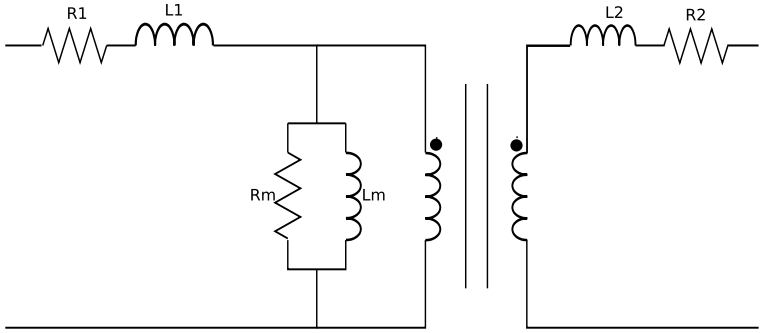

Figure 2.4: Transformer mono-phase equivalent circuit

where $\omega_b = 2\pi f_b$.

2.5.3 Transmission Lines

Similar to systems that have spatial and temporal variations, the transmission lines can be modeled using distributed or lumped parameters models. In the case of benchmark system described in this work, the transmission lines are modeled using a lumped parameters model, specifically the RL series circuit model as shown in the figure 2.5.

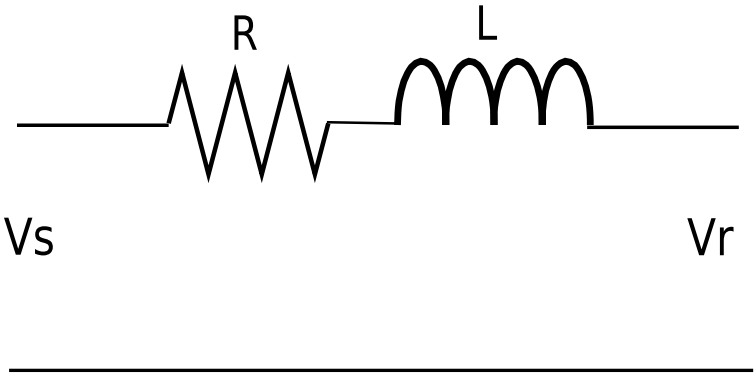

Figure 2.5: Transmission lines equivalent circuit

were V_s and V_r are the voltages at the ends of the transmission lines.

The dynamic model that depicts the voltage behavior in each phase of the transmission line is given by 2.13 to 2.15

$$\dot{i}_a = \frac{1}{L_a}(V_{sa} - V_{ra} - R_a i_a) \tag{2.13}$$

$$\dot{i}_b = \frac{1}{L_b}(V_{sb} - V_{rb} - R_b i_b) \tag{2.14}$$

$$\dot{i}_c = \frac{1}{L_c}(V_{sc} - V_{rc} - R_c i_c) \tag{2.15}$$

This model can be applied when the longitude of lines are less than 120 miles, and includes loses due to Joule effect and those associated with active and reactive power delivery.

2.5.4 Simulation Results

The benchmark system depicted before, was simulated using the software MATLAB/SIMULINK, specifically Simpower Systems toolbox. The integration routine used was ODE23 stiff/trapezoidal, with variable step. Electric power system initial conditions were found solving a load flow. The field voltage, the mechanical power and the speed of the generation units in absence of disturbances, are displayed in the figures 2.6 to 2.8, respectively.

In figures 2.6 and 2.7 it is possible to see that performance of control systems of generation units is acceptable, leading field voltages and mechanical powers of each generation unit to stable values. According to behavior exhibited in figures 2.6 and 2.7 it is possible to conclude that terminal voltage and, active and reactive power takes a constant value in steady state.

Moreover, speed of generation units reveals some aspects of the ”healthy” of the system. In figure 2.8 speed of all generation units is presented. It is possible to see that all machines speed converges to the same value. This means that all machines in the system are in synchronism and the frequency in the whole system is the same. Also there is a balance between generation and demand.

As soon as possible, the benchmark system will be implemented using only Matlab functions.

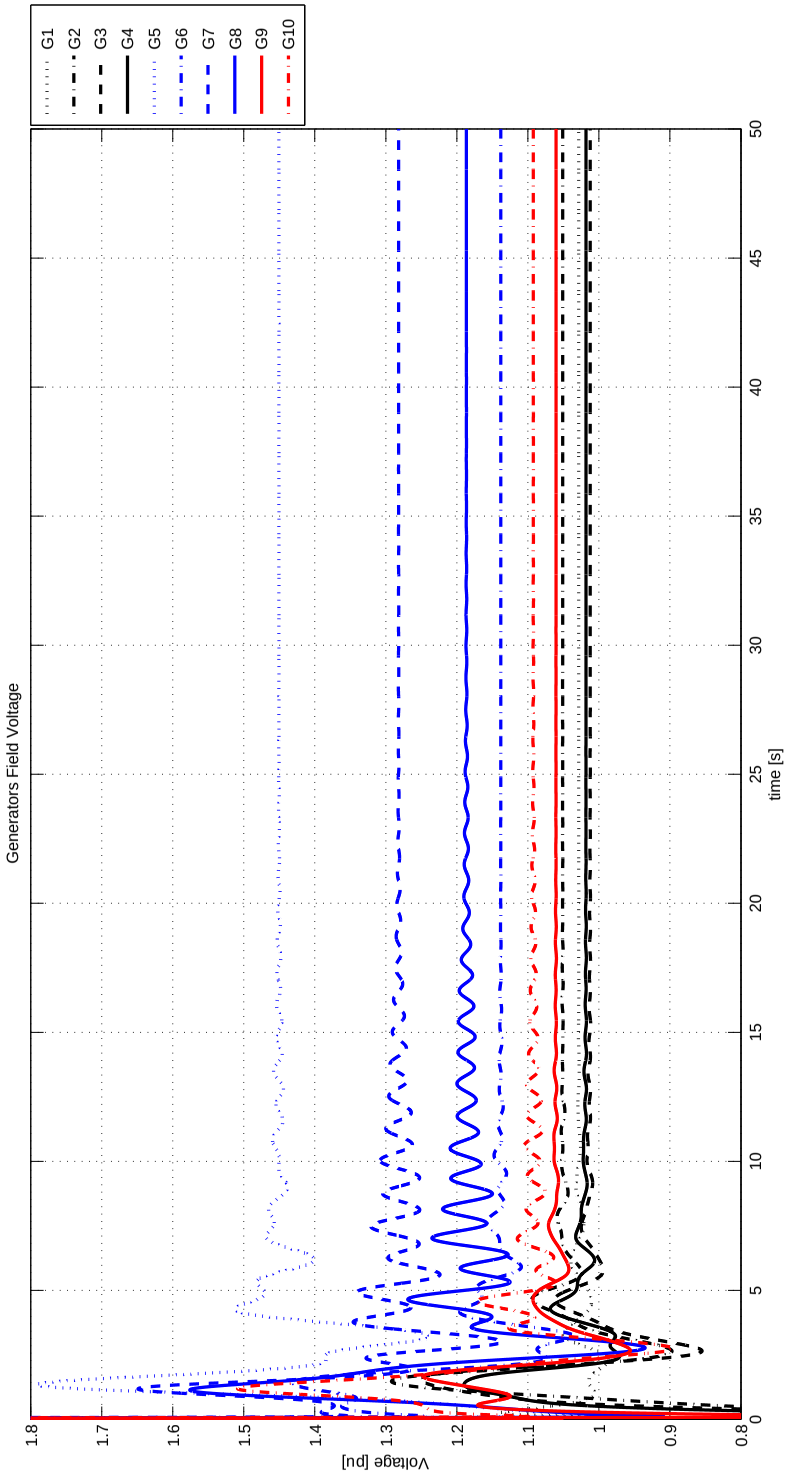

Figure 2.6: Generation units field voltage

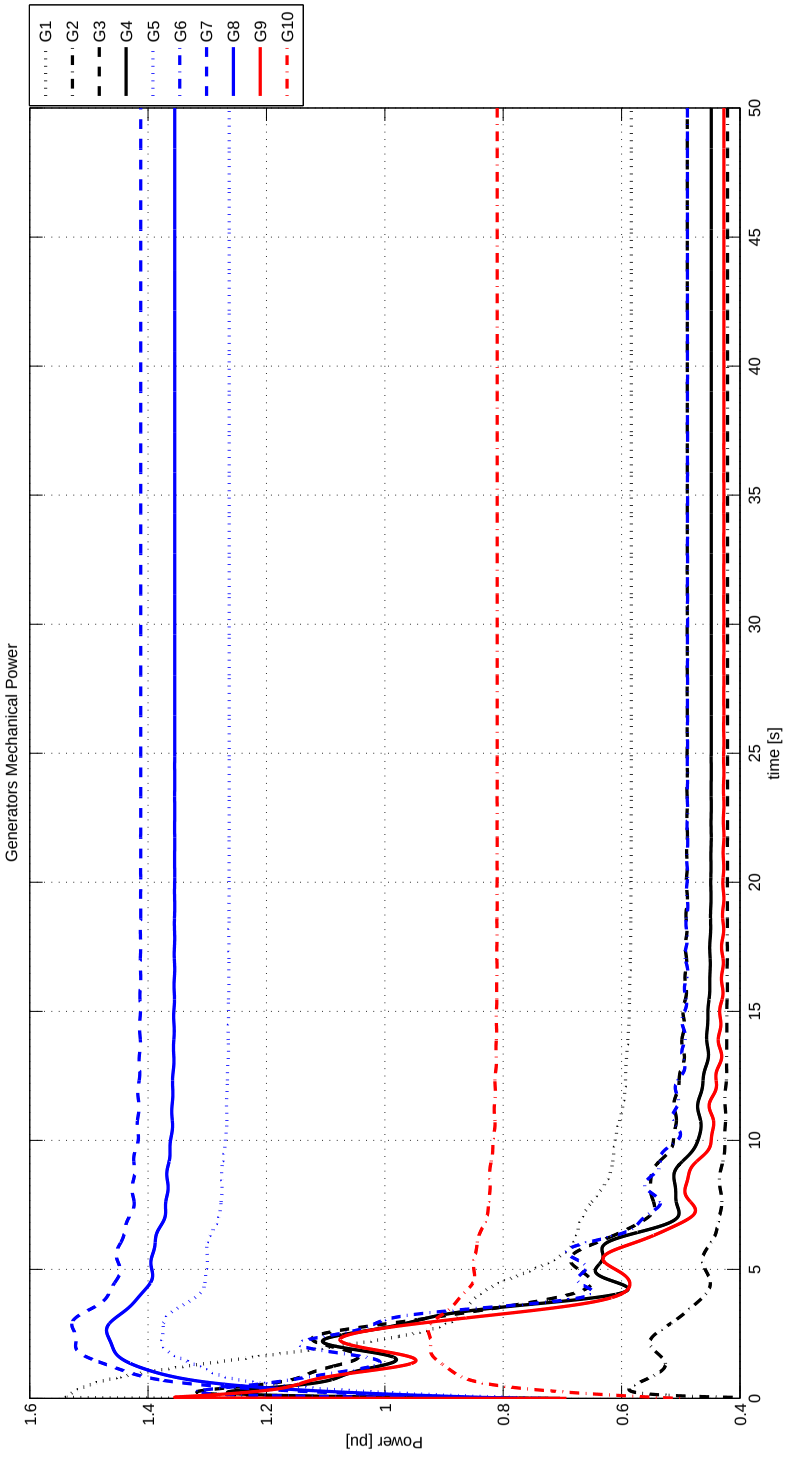

Figure 2.7: Generation units mechanical power

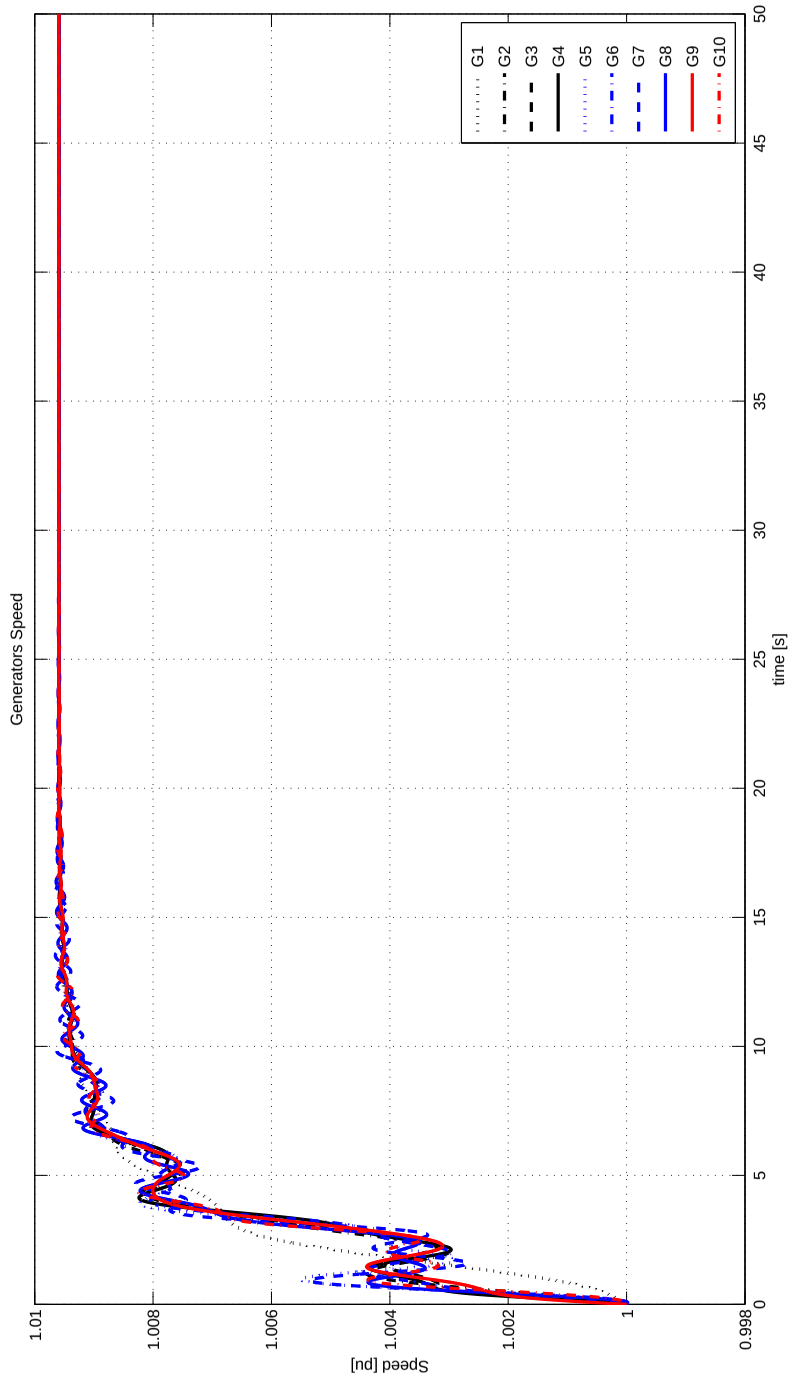

Figure 2.8: Generation units speed

Chapter 3

A Benchmark for Distributed Estimation and Control: A Generic Model for the Heat Conduction and Convection in a Rod, a Plate, and a Cube

3.1 Introduction

As it is known, reliable benchmarks are needed to test all the control and estimation strategies designed in the HD-MPC project. In this case an straightforward convection and conduction model is constructed, programmed and tested in order to offer an alternative to test some strategies of estimation and control of large-scale systems. One of the main features of this model is the possibility to simulate the heat dynamics taking into account one, two or three dimensions. This gives a heat convection and conduction model in a rod (one dimension), a plate (two dimensions), and a cube (three dimensions).

In this note a complete description of the systems is made. First, the processes are described, also the processes variables for control purposes. Then dynamic information of the systems is discussed. Also, reviewed bibliography about the processes and the applied control strategies are listed. Later, control and instrumentation issues are discussed. The note is finalized with the description of the models and the applied transformations to get numerical solutions. State-space models are presented for each process.

3.2 Description of the System

In this report, heat conduction and convection at three elements are considered: a rod, a plate, and a cube. The elements are characterized by its parameters. Therefore, conduction and convection properties become dependent of the physical composition of such elements.

3.2.1 The Process

As it was exposed earlier, the process of interest is one of three elements (a rod, a plate or a cube), some actuators and some measurement devices. Each of them is dependent of its parameters and also

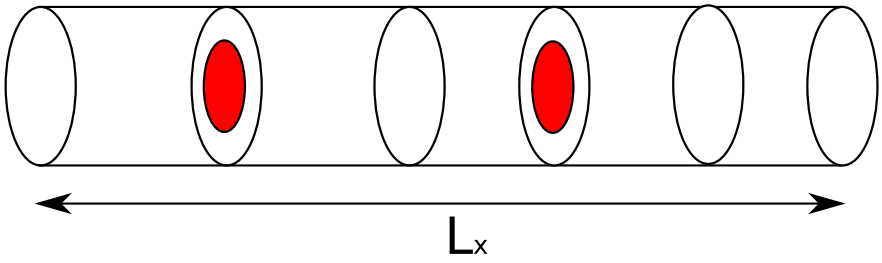

Figure 3.1: A Rod. One dimensional heat exchange.

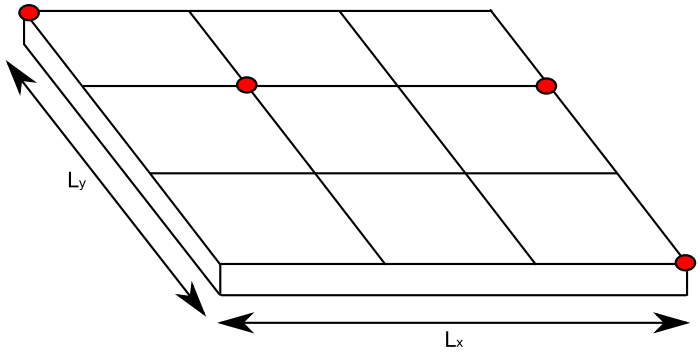

Figure 3.2: A Plate. Two dimensional heat exchange.

of its spatial configuration. The functionality of the process is focused on guarantee a temperature profile across the solid element using certain resistances (heaters) configured in some way.

Physical description

First, consider a solid rod of some material with a length L_x as in Fig. 3.1. In this case, heat conduction and convection phenomena arise throughout the rod mainly at x-axis. It can be explained, taking into account any infinitesimal element of the rod: if this infinitesimal element is at the ends of the rod, then conduction is given with the adjacent element and the convection is given between the exposed area and the environment. When the infinitesimal element is inside the rod, then the two adjacent infinitesimal elements allows the heat conduction, while convection is given between the environment and the exposed area. Finally, the configuration of the "heat points" in the rod are represented by red points in the Fig. 3.1.

A second element to be considered is a plate of dimensions L_x, and L_y (See Fig. 3.2). In this case one dimension is added to the last example. Conduction is given at any partition with the adjacent elements at two axis (x-axis and y-axis). On the other hand, there is a heat exchange by convection in all the plate on the outside surface. The red points show a given configuration of the actuators (heaters).

The third case is a three dimensional heat exchange, as it can be seen at Fig 3.3. In this case, the element is a cube of dimensions L_x, L_y, and L_z. The main idea is to obtain a model taking into account the conduction phenomena across the three dimensions, and the convection only at the outside surface. Again, the red points are a given configuration of the actuators in the system.

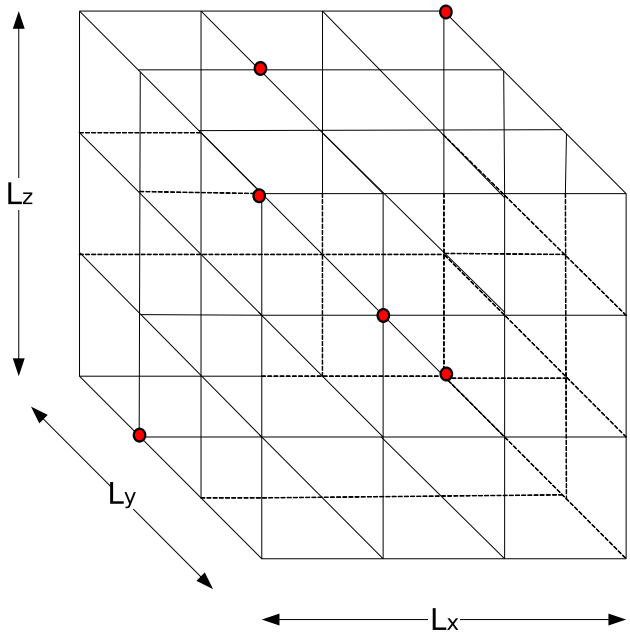

Figure 3.3: A Plate. Two dimensional heat exchange.

As it will be seen later, the phenomena-based model of the presented elements are governed by partial PDEs obtained from some energy balances. These PDEs are commonly solved with some finite differences approach in a numerical way. Then each solid element is represented by a finite number of infinitesimal elements. Therefore, the global system is composed by a finite number of interconnected subsystems, each of them with a heat exchange dynamic.

The presented cases are a starting point to implement distributed control and estimation strategies, because of the complexity of the interconnected system can be increased as desired. Computational issues such as computational effort, communication between agents, and parallelization schemes arise. On the other hand, other important issues like hierarchical control, disturbances rejection, and robustness can also be examined.

Process Variables

In this point, a classification of the process variables is made in order to establish its role inside the control and estimation problem.

Set point can be a constant reference or a path. In the present case, the temperature at each partition must be controlled in order to follow certain temperature profile or simply a constant reference. The size of the vector of controlled variables is dependent of the number of partitions made at each dimension in each problem. The constraints over the controlled variables are imposed by the phenomenology, that is, the temperature can not be lower than the environment temperature and greater than a saturation temperature. This temperature dependents on the heat injected to the system.

The control goals are reached through the manipulated variables. In this case, an actuator (heater) can be placed at each infinitesimal element. Therefore, a heater can be located at each partition as desired. Each heater gives energy to the system as power per unit of length (in the rod), power per unit of area (in the plate), and power per unit of volume (in the cube). The constraints on the manipulated

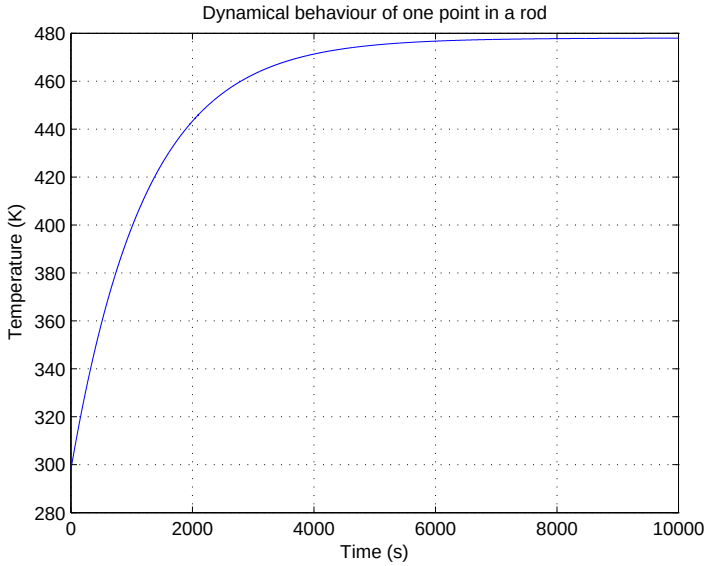

Figure 3.4: Time response of one point in the rod

variables are a lower and an upper saturation, because of the injected power must be greater to zero and smaller than a limited power value.

The output of the system is given by the measured variables. In this case all the states can be measured, but as it is known if there exist more partitions, there are less possibility to physically install the measure devices. The main idea is to install temperature sensors in order to get the observability of the system. This sensor location can be made in an optimal way. Then, any observer can be designed preferably in a distributed procedure.

There also exist certain unknown inputs to the system, those are, the process disturbances. These variables can be model parameters and inputs from other subsystems. Only the environment temperature can be measured. Other model parameters can change as a function of the temperature in an unknown way. Finally, local control in any subsystem can influence the dynamic behavior of the neighboring subsystems.

Dynamic information

As it will be shown later, each system can be described with a PDE. This PDE can be discretized in space and time. When the PDE only is discretized in space a linear ordinary differential equation (ODE) is obtained at each partition. The behavior of any ODE is described by a first order dynamic. The time response is approximately 1200 $[s]$ as it can be seen from Fig. 3.4, when a step signal is applied. The initial value of the step was zero and after an excitation time this value is switched to a maximum power value.

Process bibliography

Process phenomena can be easily found at any book of heat transfer [Incropera and de Witt, 2001], [Lienhard and Lienhard, 2008]. As an example in [Bird et al, 2007] there are many study cases related

to the heat transfer in a rigid body. All of them show the associated phenomenology, but the numerical solution of the PDE is not explicit.

In [Rapson, 2008] a model of the conduction and convection through a rod is shown. This model is based on a PDE and it is solved with a finite differences scheme. A state-space model is derived. Some centralized control strategies are applied to the study case like LQR (Linear Quadratic Regulator), PI (Proportional-Integral) controller, $\mathscr{H}_\infty$ Control, and MPC (Model Predictive Control). Also, some decentralized control schemes are tested.

An MPC based on a reduced model using proper orthogonal decomposition (POD) is applied to control the temperature profile in a bidimensional plate, [Márquez, 2007].

3.2.2 Control System and Instrumentation

As it was shown earlier, the processes are available only in simulation. Therefore, any modification can be made on the manipulated variables, measured variables, and disturbances, in order to get a complex process as desired.

Distributed control

In [Rapson, 2008] a simple partition is proposed to obtain a decentralized control of the rod. The idea is to break down the system into a series of SISO systems, such that any controller receives information from and has control over one node. All the proposed control strategies are decentralized, but these are not distributed. Therefore communication issues did not arise.

Sensors and actuators

Only temperature sensors (thermocouples) are assumed at each case. The challenge is focused on finding the minimal number of sensors and its locations in order to guarantee the system observability and then estimation of the whole state of the system. The actuators are heaters (resistances) with a predefined range of operation. Numerical values of these parameters are shown later.

3.2.3 Security

No security issues are concerned here.

3.2.4 Process Models

In the present report only models based on the phenomenology of the exposed processes are discussed.

Rod Model

Consider a solid rod as in 3.1. Assuming a uniform composition of the material, and the heat phenomena are given predominantly at one axis (x-axis), then an energy balance can be made to an internal slice, as it is shown in Fig. 3.5:

$$\rho C_p (A_T \partial x) \frac{\partial T}{\partial t} = \left(\kappa \frac{\partial^2 T}{\partial x^2} + g(x,t) \right) A_T \partial x \tag{3.1}$$

where ρ is the density of the rod, C_p is the heat capacity per unit of mass, κ is the thermal conductivity, P is the perimeter of the cross-sectional circumference, ∂x is the width of the slice,

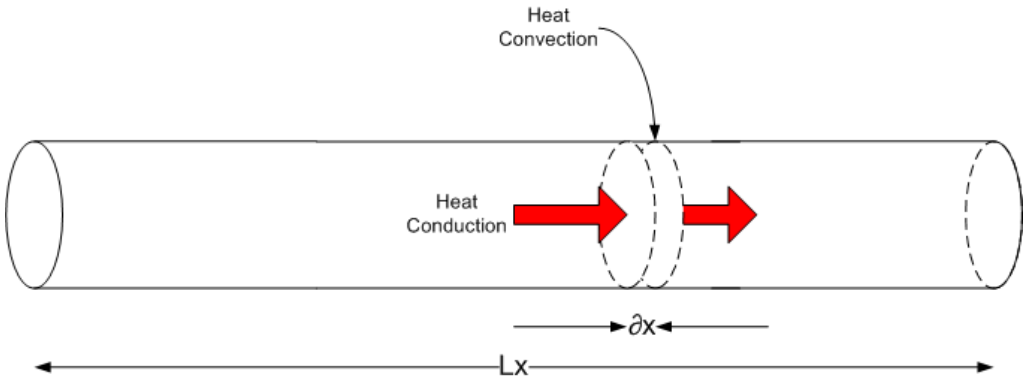

Figure 3.5: Modeling a slice of a rod.

$A_T = P\partial x$ is the area exposed to the environment, T is the temperature inside the slice, x and t are the spatial and temporal variables, and $g(x,t)$ is a generation function defined as:

$$g(x,t)A_T\partial x = \dot{Q}(x,t)P\partial x + h(T_{env} - T(x,t))P\partial x \tag{3.2}$$

with $\dot{Q}(x,t)$ the heater power per unit of area, h the convection coefficient, and T_{env} the temperature at the environment.

Then, replacing (3.2) in (3.1), a final PDE is obtained:

$$\frac{\partial T}{\partial t} = \frac{1}{\rho C_p}\left[\kappa\frac{\partial^2 T}{\partial x^2} + \frac{P}{A_T}\dot{Q}(x,t) + \frac{hP}{A_T}(T_{env} - T)\right] \tag{3.3}$$

To model the ends of the rod, an additional assumption must be made: the conduction phenomena is only given in one direction. The boundary conditions are further explained later.

Plate model

The plate model is quite similar to the rod. Also similar assumptions must be made in order to get a phenomenological model: uniform composition of the plate, there are heat conduction at the two axis (x-axis and y-axis), and heat exchange by convection is given at the exposed surface. Fig. 3.6 shows a plate with differential dimensions, where the energy balance is applied. The model is given by Eq. (3.4).

$$\frac{\partial T}{\partial t} = \frac{1}{\rho C_p}\left[\kappa\frac{\partial^2 T}{\partial x^2} + \kappa\frac{\partial^2 T}{\partial y^2} + \frac{\dot{Q}_s(x,y,t)}{\partial z} + \frac{h}{\partial z}(T_{env} - T)\right] \tag{3.4}$$

where x and y are the spatial variables as it is indicated in the Fig. 3.6, $\dot{Q}_s$ is the heat power per unit of area, and the other variables are as in the rod model.

Cube model

The three dimensional heat exchange model of a solid element like a cube is similar to the previous models. As additional assumptions it can be stated that convection is only given at external surfaces (at one, two or three dimensions in accordance with the exposed surfaces of the differential element),

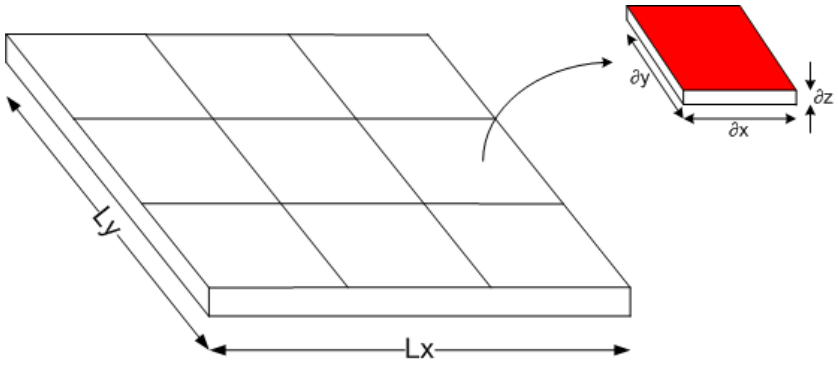

Figure 3.6: Modeling a differential plate.

then internal points do not experiment this phenomenon. On the other hand, conduction phenomena is given at the three dimensions. Therefore the general model of the heat exchange in an internal differential cube can be extended as:

$$\begin{aligned}\frac{\partial T}{\partial t} = \frac{1}{\rho C_p}\Big[&\kappa\frac{\partial^2 T}{\partial x^2} + \kappa\frac{\partial^2 T}{\partial y^2} + \kappa\frac{\partial^2 T}{\partial z^2} + \dot{Q}_v \\ &+h\left(\frac{1}{\partial x} + \frac{1}{\partial y} + \frac{1}{\partial z}\right)(T_{env} - T)\Big]\end{aligned} \tag{3.5}$$

where x, y, and z are the spatial variables, and $\dot{Q}_v = \dot{Q}_v(x,y,z,t)$ is a heat power per unit of volume. In Fig. 3.7 a volumetric differential element is shown.

To model the physical boundaries of the rod, the plate, and the cube, additional assumptions must be made. The boundary conditions are further explained later.

Model discretization

In order to solve numerically the previous models, it is mandatory to approximate the partial derivatives. A straightforward way to do this is applying the finite differences method. This method uses an approximation of the derivatives based on a truncation of the Taylor series at the first order term. Then a derivative can be expressed as:

$$\frac{\partial u}{\partial h} \approx \frac{u_{i+1} - u_i}{\Delta h} \tag{3.6}$$

$$\frac{\partial u}{\partial h} \approx \frac{u_i - u_{i-1}}{\Delta h} \tag{3.7}$$

$$\frac{\partial u}{\partial h} \approx \frac{u_{i+1} - u_{i-1}}{2\Delta h} \tag{3.8}$$

where i is the discretization variable. Eqs. (3.6), (3.7), and (3.8) are known as forward, backward, and central approximations of the derivative respectively. If the previous approximations are successively applied, then second order derivatives can be found as:

$$\frac{\partial^2 u}{\partial h^2} \approx \frac{u_{i+2} - 2u_{i-1} + u_i}{\Delta h^2} \tag{3.9}$$

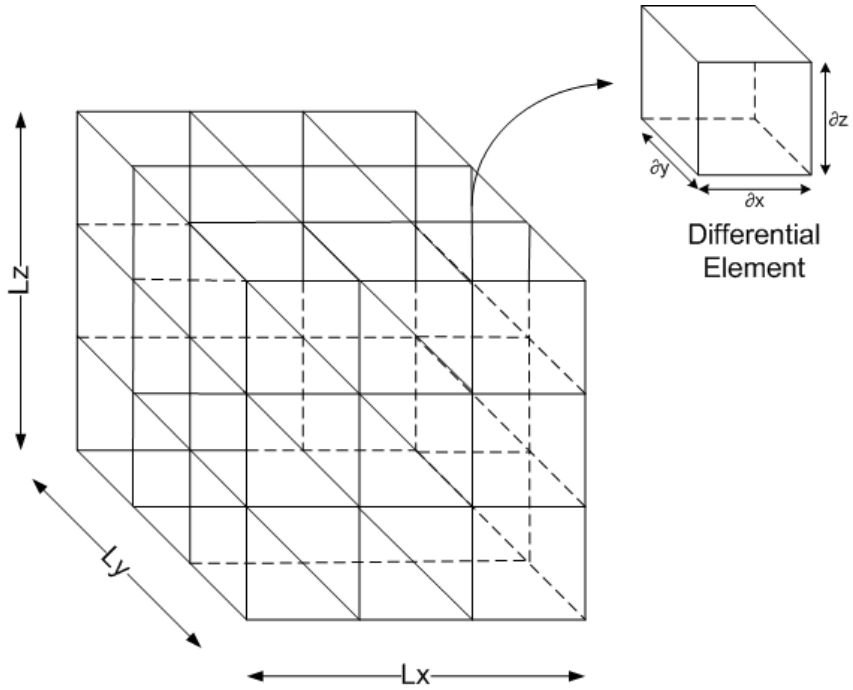

Figure 3.7: Heat exchange in a generic differential cube.

$$\frac{\partial^2 u}{\partial h^2} \approx \frac{u_i - 2u_{i-1} + u_{i-2}}{\Delta h^2} \tag{3.10}$$

$$\frac{\partial^2 u}{\partial h^2} \approx \frac{u_{i+1} - 2u_i + u_{i-1}}{\Delta h^2} \tag{3.11}$$

If the approximations are applied to the rod equation, then three equations must be considered:

$$\frac{dT_i}{dt} = \frac{1}{\rho C_p}\left[\kappa\frac{T_{i+1} - 2T_i + T_{i-1}}{\Delta x^2} + \frac{P}{A_T}\dot{Q}_i + \frac{hP}{A_T}\left(T_{env} - T_i\right)\right] \tag{3.12}$$

$$\frac{dT_i}{dt} = \frac{1}{\rho C_p}\left[\kappa\frac{T_{i+1} - T_i}{\Delta x} + \frac{P}{A_T}\dot{Q}_i + \frac{hP}{A_T}\left(T_{env} - T_i\right)\right] \tag{3.13}$$

$$\frac{dT_i}{dt} = \frac{1}{\rho C_p}\left[\kappa\frac{T_{i-1} - T_i}{\Delta x} + \frac{P}{A_T}\dot{Q}_i + \frac{hP}{A_T}\left(T_{env} - T_i\right)\right] \tag{3.14}$$

with i the discretization variable at x-axis, and Δx the length of the differential spatial partition. Eqs. (3.13), and (3.14) are applied at the ends of the rod. Eq. (3.12) is applied inside the rod, when the slice is between two neighboring slices. The number of equations are the same as partitions considered in the rod.

Similar assumptions are made to apply the approximations in the plate. At the physical left boundary, and at the physical right boundary a first order forward and backward approximations are applied at x-variable respectively. In a similar way, at the lower and upper physical boundaries a first order forward and backward approximations are applied at y-variable respectively. Otherwise second order

central approximations are applied to the interior points. Finally both forward or backward approximations must be made at the corners. The general interior point equation is written as:

$$\frac{dT_{(i,j)}}{dt} = \frac{1}{\rho C_p}\left[\kappa\frac{T_{(i+1,j)}-2T_{(i,j)}+T_{(i-1,j)}}{\Delta x^2} + \kappa\frac{T_{(i,j+1)}-2T_{(i,j)}+T_{(i,j-1)}}{\Delta y^2} + \frac{\dot{Q}_{s(i,j)}}{\Delta z} + \frac{h}{\Delta z}\left(T_{env} - T_{(i,j)}\right)\right] \tag{3.15}$$

with i, j the discretization variables, and Δx, Δy the length of the spatial partitions at the x and y axis respectively. The number of equations are depending of the number of partitions assumed at each axis. Then a set of $N_x \cdot N_y$ ODEs is generated, where N_x and N_y are the number of nodes at x-axis and y-axis respectively.

Finally, Eq. (3.5) can be transformed into a set of ODEs using the same approximations at three axes. Instead of points or lines as physical boundaries, the physical boundaries are each cube face. Hence, forward and backward approximations to the x-derivative are used at the left and right faces respectively. Also forward and backward approximations to the y-derivative are used at the front and back faces respectively. Finally, to approximate the z-derivatives at the lower and upper faces forward and backward approximations are respectively used. A combination of approximations must be applied at the corner points, i.e the point (1,1,1) has forward approximations at each variable. Otherwise, central approximations are applied to internal points in the cube. The general equation for an interior point is as follows:

$$\frac{dT_{(i,j,k)}}{dt} = \frac{1}{\rho C_p}\left[\kappa\frac{T_{(i+1,j,k)}-2T_{(i,j,k)}+T_{(i-1,j,k)}}{\Delta x^2} + \kappa\frac{T_{(i,j+1,k)}-2T_{(i,j,k)}+T_{(i,j-1,k)}}{\Delta y^2} \kappa\frac{T_{(i,j,k+1)}-2T_{(i,j,k)}+T_{(i,j,k-1)}}{\Delta z^2} + \dot{Q}_{c(i,j,k)} + h\left(\frac{1}{\Delta x}+\frac{1}{\Delta y}+\frac{1}{\Delta z}\right)\left(T_{env} - T_{(i,j,k)}\right)\right] \tag{3.16}$$

with i, j, k the discretization variables, and Δx, Δy, Δz the length of the spatial partitions at the x, y and z axis respectively. $\dot{Q}_c$ is the heat power per volume unit. The number of equations depends of the number of partitions assumed at each axis. Then a set of $N_x \cdot N_y \cdot N_z$ ODEs is generated, where N_z are the number of nodes at z-axis.

State-Space models

Consider a generic state-space model:

$$\begin{aligned} \dot{x} &= Ax + Bu_a \\ y &= Cx \end{aligned} \tag{3.17}$$

where x is the state of the system, u_a is the input, and y is the output or measured variables. Moreover $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, and $C \in \mathbb{R}^{n\times p}$, with n is the number of states, m is the number of inputs, and p is the number of outputs.

As the new models have finite dimension due to the discretization of the PDEs, a state-space framework can be used to represent the entire model of each physical system.

Consider first a possible discretization of a rod as in the Fig (3.8). The model of the rod can be written as Eq. (3.17), using Eqs. (3.12) to (3.15):

$$A_R = \frac{\kappa}{\rho C_p \Delta x}\begin{bmatrix} -1 & 1 & & & 0 \\ 1 & -2 & 1 & & \\ & & \ddots & & \\ & & 1 & -2 & 1 \\ 0 & & & 1 & -1 \end{bmatrix} - \frac{hP}{\rho C_p A_T}\begin{bmatrix} 1 & & & & 0 \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ 0 & & & & 1 \end{bmatrix} \tag{3.18}$$

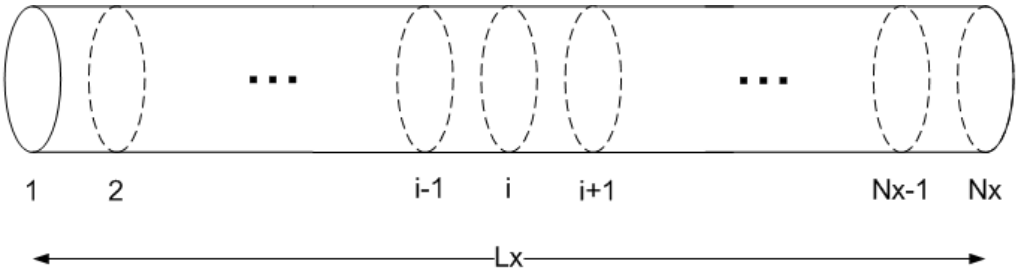

Figure 3.8: Spatial discretization in a rod.

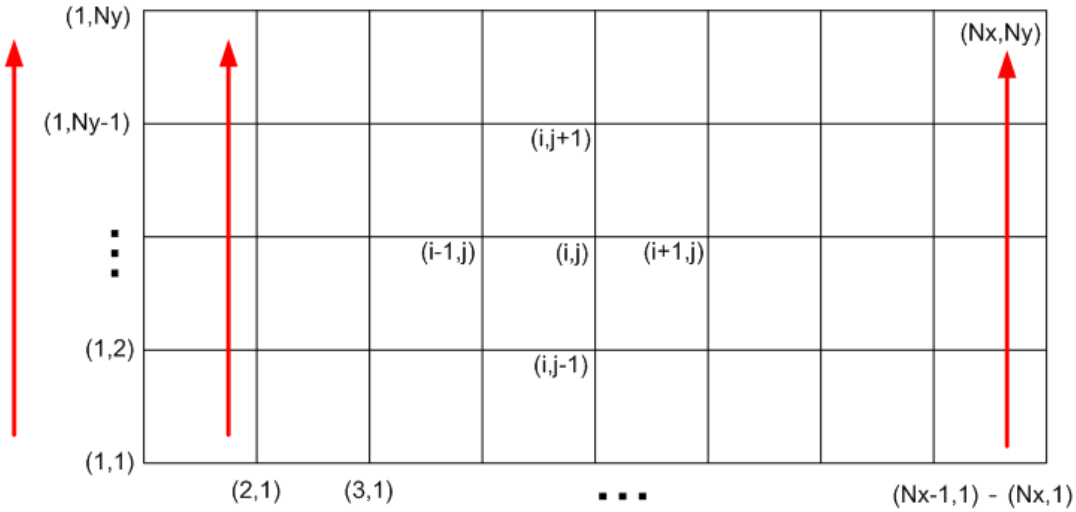

Figure 3.9: Spatial discretization in a plate.

$$B_R = \begin{bmatrix} \frac{P}{\rho C_p A_T} & 0 & \cdots & & 0 & \vdots & \frac{hP}{\rho C_p A_T} \\ 0 & \frac{P}{\rho C_p A_T} & & & \vdots & \vdots & \frac{hP}{\rho C_p A_T} \\ \vdots & & \ddots & & & \vdots & \vdots \\ & & & \frac{P}{\rho C_p A_T} & 0 & \vdots & \frac{hP}{\rho C_p A_T} \\ 0 & \cdots & & 0 & \frac{P}{\rho C_p A_T} & \vdots & \frac{hP}{\rho C_p A_T} \end{bmatrix} \tag{3.19}$$

with $x = \begin{bmatrix} T_1 & T_2 & & T_{N_x-1} & T_{N_x} \end{bmatrix}^T$, $u_a = \begin{bmatrix} q_1 & q_2 & \cdots & q_{N_x-1} & q_{N_x} & T_{env} \end{bmatrix}$, $q_1, q_2, ..., q_{N_x-1}$, and q_{N_x} the heat power applied to each resistance or actuator, A_R, and B_R the rod matrices. C_c depends on the number and location of measured states.

In order to get the state space model of the plate, it must be taken into account the ordering of the equations. In this note, the equations follow the order shown at Fig. (3.9). Note that, the first equation is concerned with $\dot{T}_{1,1}$, the second equation with $\dot{T}_{1,2}$, until $\dot{T}_{1,N_y}$. Then the equation of $\dot{T}_{2,1}$ follows. The order is repeated from the lower point to the upper point at each column. Eqs. (3.20) and (3.21)

show the state and input matrices for the plate:

$$A_p = \begin{bmatrix}
a & \Delta yC_2 & & \Delta xC_1 & 0 & & & & \cdots & & & & 0 \\
C_2 & b & C_2 & & & & & & & & & & \\
& \ddots & \ddots & \ddots & & \ddots & & & & & & & \\
C_1 & & \Delta yC_2 & a & 0 & & & & & & & & \vdots \\
0 & & & 0 & c & \Delta yC_2 & & \Delta xC_1 & \ddots & & & & \\
& & \ddots & & C_2 & d & C_2 & & C_1 & & & & \\
& & & & & \ddots & \ddots & \ddots & & & & & \\
& & & & C_1 & & \Delta_y C_2 & c & 0 & & \ddots & & \\
\vdots & & & & \ddots & \Delta yC_1 & & 0 & \ddots & \Delta yC_2 & & & 0 \\
& & & & & & & & C_2 & a & C_2 & & C_1 \\
& & & & & & & \ddots & & \ddots & b & \ddots & \\
& & & & & & & & & & & \ddots & 0 \\
0 & & & & \cdots & & & & 0 & \Delta yC_1 & & \Delta yC_2 & a
\end{bmatrix} \tag{3.20}$$

$$B_p = \left[\begin{array}{cccccc|c}
\frac{1}{\rho C_p \Delta z} & 0 & \cdots & & 0 & & \frac{h}{\rho C_p \Delta z} \\
0 & \frac{1}{\rho C_p \Delta z} & \ddots & & \vdots & & \frac{h}{\rho C_p \Delta z} \\
\vdots & \ddots & \ddots & & & & \vdots \\
& & & \frac{1}{\rho C_p \Delta z} & 0 & & \frac{h}{\rho C_p \Delta z} \\
0 & \cdots & & 0 & \frac{1}{\rho C_p \Delta z} & & \frac{h}{\rho C_p \Delta z}
\end{array}\right] \tag{3.21}$$

A_p, and B_p the state and input matrices of the plate. C_p depends on the number of measured states. Note from Eq. (3.17) and Fig. (3.9) the state and input vectors must be:

$$x = \begin{bmatrix} T_{1,1} & T_{1,2} & \cdots & T_{1,N_y-1} & T_{1,N_y} & T_{2,1} & \cdots & T_{N_x,1} & \cdots & T_{N_x,N_y} \end{bmatrix}^T \tag{3.22}$$

$$u_a = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{N_xN_y-1} & q_{N_xN_y} & T_{env} \end{bmatrix} \tag{3.23}$$

with q_{11}, q_{12}, ..., $q_{N_xN_y-1}$, and $q_{N_xN_y}$ the heat power per area unit applied to each resistance or actuator.

Also, by simplicity some variables are defined:

$$C_1 = \frac{\kappa}{\rho C_p \Delta x^2}, C_2 = \frac{\kappa}{\rho C_p \Delta y^2}, C_3 = \frac{1}{\rho C_p \Delta z}, C_4 = \frac{h}{\rho C_p \Delta z} \tag{3.24}$$

$$a = -\Delta_x C_1 - \Delta_y C_2 - C_4 \tag{3.25}$$

$$b = -2C_1 - \Delta_y C_2 - C_4 \tag{3.26}$$

$$c = -\Delta_x C_1 - 2C_2 - C_4 \tag{3.27}$$

$$d = -2C_1 - 2C_2 - C_4 \tag{3.28}$$

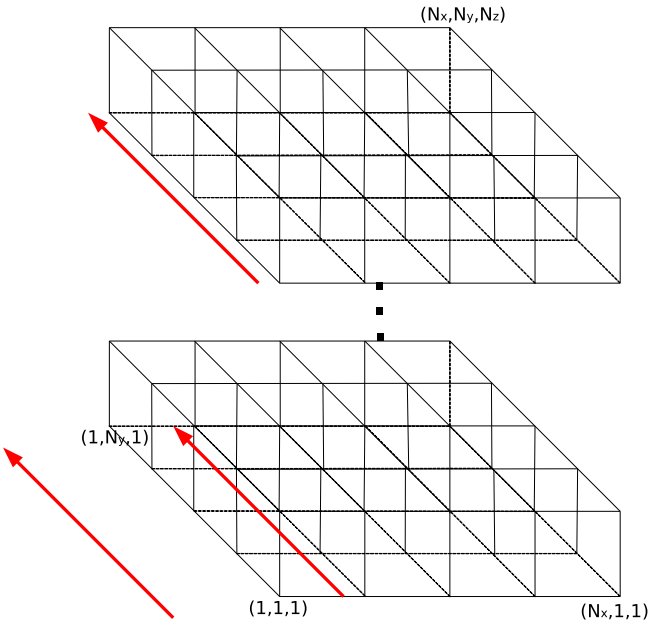

Figure 3.10: Spatial discretization in a cube.

About the matrix A_p, note the leading diagonal pattern: first as corner condition the entry is a, then following the physical left boundary of the plate the entries are b until the first upper corner when the diagonal entry is also a. Then, the lower and upper physical boundaries are filled with c as entries of the leading diagonal. Finally the left physical boundary pattern is repeated at the right physical boundary. Moreover the other diagonals have recognizable patterns: the following down diagonal is filled first with C_2, and $\Delta_y C_2$ many times as internal points between the lower and upper physical boundaries. Finally, a zero entry (0) is placed at the upper physical boundary; the next diagonal above the leading diagonal has the same pattern of the last diagonal, but exchanging $\Delta_y C_2$ by C_2. Finally, there are two diagonals starting from the points $(1, N_y+1)$ and $(N_y+1, 1)$. The first one are filled with C_1, and the last N_y entries are filled with $\Delta_x C_1$. Conversely, the other diagonal starts with N_y entries as $\Delta_x C_1$ and the remaining entries are filled with C_1.

In order to generalize the heat exchange problem at three dimensions, a state-space model must be found for the cube problem. As in the previous examples, consider a possible discretization of the cube by means of Fig. (3.10). It is assumed a number of plates one above the other. Therefore, the discretization is as in the plate, but taking into account the variation a the z-variable.

Eqs (3.29) and (3.30) show the state and input matrices for the cube:

$$A_c = \begin{bmatrix} A_{p1} & \Delta z C c_3 & & & & & \\ C c_3 & A_{p2} & \ddots & & & 0 & \\ & \ddots & & \Delta z C c_3 & & & \\ & & C c_3 & \ddots & C c_3 & & \\ & & & \Delta z C c_3 & & & \\ & 0 & & & \ddots & A_{p2} & \ddots \\ & & & & & \Delta z C c_3 & A_{p1} & C c_3 \end{bmatrix} \tag{3.29}$$

$$B_c = \left[\begin{array}{cccc|c} \frac{1}{\rho C_p} & & & & \\ & \frac{1}{\rho C_p} & & 0 & \\ & & \ddots & & \frac{h}{\rho C_p} CT \\ & 0 & & \frac{1}{\rho C_p} & \\ & & & & \frac{1}{\rho C_p} \end{array}\right] \tag{3.30}$$

A_c, and B_c are the state and input matrices of the cube. C_c depends on the number and location of the measured states. Note from Eq. (3.17) and Fig. (3.10) the state and input vectors must be:

$$\begin{aligned} x = \big[\ T_{1,1,1} \quad T_{1,2,1} \quad \cdots \quad T_{1,N_y-1,1} \quad T_{1,N_y,1} \quad T_{2,1,1} \quad \cdots \quad T_{N_x,1,1} \quad \cdots \quad T_{N_x,N_y,1} \\ T_{1,1,2} \quad \cdots \quad T_{N_x,N_y,2} \quad \cdots \quad T_{1,1,N_z} \quad \cdots \quad T_{N_x,N_y,N_z} \ \big]^T \end{aligned} \tag{3.31}$$

$$u_a = \big[\ q_1 \quad q_2 \quad \cdots \quad q_{N_xN_yN_z-1} \quad q_{N_xN_yN_z} \quad T_{env} \ \big] \tag{3.32}$$

with q_{111}, q_{121}, ..., $q_{N_xN_yN_z-1}$, and $q_{N_xN_yN_z}$ the heat power per volume unit applied to each resistance or actuator. Finally, in order to define the matrix B_c is mandatory to declare the term vector $CT \in \mathbb{R}^{N_xN_yN_z}$. This is a convection term and it is dependent of the matrix order:

$$CT = \big[\ \delta_{xyz} \quad \delta_{xz} \quad \cdots \quad \delta_{xyz} \quad \delta_{xy} \quad \delta_x \quad \cdots \quad \delta_{xy} \quad \cdots \quad \delta_{xyz} \quad \delta_{xz} \quad \cdots \quad \delta_{xyz} \ \big]^T \tag{3.33}$$

where

$$\delta_x = \frac{1}{\Delta x},\ \delta_y = \frac{1}{\Delta y},\ \delta_z = \frac{1}{\Delta z},\ \delta_{xy} = \frac{1}{\Delta x} + \frac{1}{\Delta y},\ \delta_{xz} = \frac{1}{\Delta x} + \frac{1}{\Delta z} \tag{3.34}$$

$$\delta_{yz} = \frac{1}{\Delta y} + \frac{1}{\Delta z}, \delta_{xyz} = \frac{1}{\Delta x} + \frac{1}{\Delta y} + \frac{1}{\Delta z} \tag{3.35}$$

The leading diagonal of the matrix (3.29) is composed by all the plates matrices as in Eq. (3.20). The lower and upper plates of the cube are represented by A_{p1}, while the intermediate plates are represented by A_{p2}. These matrices are as follows:

$$A_{p1} = \begin{bmatrix} a_c & \Delta_y Cc_2 & & \Delta_x Cc_1 & 0 & & & & \cdots & & & 0 \\ Cc_2 & b_c & Cc_2 & & & & & & & & & \\ & \Delta_y Cc_2 & \ddots & \ddots & & \ddots & & & & & & \\ Cc_1 & & \ddots & a_c & 0 & & & & & & & \\ 0 & & & 0 & c_c & \Delta_y Cc_2 & & \Delta_x Cc_1 & \ddots & & & \vdots \\ & & \ddots & & Cc_2 & d_c & Cc_2 & & Cc_1 & & & \\ & & & & & \Delta_y Cc_2 & \ddots & \ddots & & & & \\ & & & & Cc_1 & & \ddots & c_c & 0 & & \ddots & \\ \vdots & & & & \ddots & \Delta_x Cc_1 & & 0 & \ddots & \Delta_y Cc_2 & & 0 \\ & & & & & & & & Cc_2 & a_c & Cc_2 & Cc_1 \\ & & & & & & & \ddots & & \Delta_y Cc_2 & b_c & \ddots \\ & & & & & & & & & & \ddots & \ddots \\ 0 & & & & \cdots & & & & 0 & \Delta_x Cc_1 & & 0 \end{bmatrix} \tag{3.36}$$

$$A_{p2} = \begin{bmatrix}
e & \Delta yCc_2 & & \Delta xCc_1 & 0 & & & & \cdots & & & & 0 \\
Cc_2 & f & Cc_2 & & & & & & & & & & \\
 & \Delta yCc_2 & \ddots & & & \ddots & & & & & & & \\
Cc_1 & & \ddots & e & 0 & & & & & & & & \\
0 & & & 0 & g & \Delta yCc_2 & & \Delta xCc_1 & \ddots & & & & \vdots \\
 & & \ddots & & Cc_2 & h & Cc_2 & & Cc_1 & & & & \\
 & & & & & \Delta yCc_2 & \ddots & \ddots & & & & & \\
 & & & & Cc_1 & & \ddots & g & 0 & & \ddots & & \\
\vdots & & & & \ddots & \Delta xCc_1 & & 0 & \ddots & \Delta yCc_2 & & & 0 \\
 & & & & & & & & Cc_2 & e & Cc_2 & & Cc_1 \\
 & & & & & & & \ddots & & \Delta yCc_2 & f & \ddots & \\
 & & & & & & & & & & \ddots & \ddots & 0 \\
0 & & & & \cdots & & & & 0 & \Delta xCc_1 & & 0 & e
\end{bmatrix} \tag{3.37}$$

with:

$$Cc_1 = \frac{\kappa}{\rho C_p \Delta x^2}, Cc_2 = \frac{\kappa}{\rho C_p \Delta y^2}, Cc_3 = \frac{\kappa}{\rho C_p \Delta z^2} \tag{3.38}$$

$$Cc_4 = \frac{1}{\rho C_p}, Cc_5 = \frac{h}{\rho C_p} \tag{3.39}$$

$$a_c = -\Delta xCc_1 - \Delta yC_2 - \Delta zCc_3 - Cc_5\delta_{xyz} \tag{3.40}$$

$$b_c = -\Delta xCc_1 - 2Cc_2 - \Delta zCc_3 - Cc_5\delta_{xz} \tag{3.41}$$

$$c_c = -2Cc_1 - \Delta yCc_2 - \Delta zCc3 - Cc_5\delta_{yz} \tag{3.42}$$

$$d_c = -2Cc_1 - 2Cc_2 - \Delta zCc3 - Cc_5\delta_z \tag{3.43}$$

$$e = -\Delta xCc_1 - \Delta yCc_2 - 2Cc_3 - Cc_5\delta_{xy} \tag{3.44}$$

$$f = -\Delta xCc_1 - 2Cc_2 - 2Cc_3 - Cc_5\delta_x \tag{3.45}$$

$$g = -2Cc_1 - \Delta yCc_2 - 2Cc3 - Cc_5\delta_y \tag{3.46}$$

$$h = -2Cc_1 - 2Cc_2 - 2Cc3 \tag{3.47}$$

Following the definition of the matrix A_c, there are some complementary diagonals. From Eq. (3.29) at the position $(1, N_xN_y)$ begins a diagonal filled with ΔzCc_3 at the first (N_xN_y) entries. The other entries are filled with Cc_3. In a similar way, at $(N_xN_y, 1)$ begins a diagonal as the previous one except by the elements of ΔzCc_3 are Cc_3 and vice versa.

Model parameters

As it was exposed, model parameters are strongly dependent of the material of any element. In this case, the model parameters was taken from [Rapson, 2008], and these are concerned with EN AW1350-F Aluminum. Also, it is assumed that the element (rod, plate, or cube) is always inside an environment temperature. The parameters are presented in Table 3.1. Further details of the parameters can be found in [Rapson, 2008].

Table 3.1: Model parameters

Symbol	Parameter	Min	Nominal	Max	Units
ρ	Density	2600	2700	2800	$Kg.m^{-3}$
κ	Thermal conductivity	230	230	234	$W.m^{-1}.K^{-1}$
C_p	Heat capacity per mass unit	900	900	900	$J.K^{-1}.Kg^{-1}$
$\dot{Q}_{R\max}$	Max heat power (rod)	0	20	200	W
$\dot{Q}_{p\max}$	Max heat power (plate)	0	200	2000	$W.m^{-2}$
$\dot{Q}_{c\max}$	Max heat power (cube)	0	2000	20000	$W.m^{-3}$
h	Convection coefficient	2	10	25	$W.m^{-2}.K^{-1}$
T_{env}	Environment temperature	18	25	30	$^{\circ}C$

About model simulation

In order to solve a PDE by the presented discretization procedure, initial and boundary conditions arise. Initial conditions are assumed as the environment temperature, that is, the whole element has the same temperature of the room at the beginning of the experiment. Neumann and Robin boundary conditions are often used in this kind of equations. In this model, boundary conditions are not programmed as usual, looking for versatility. Instead, the physical boundaries are also represented by differential equations in a forward or backward approximation according to the case.

The simulations was programmed in MATLAB® and Simulink®. There are four files to perform the simulations: *paramen.txt*, *Paramfcnen.m*, *CCModelSF.mdl*, and *STcont.m*.

First modify as it is desired the parameter archive named paramen.txt. In this archive the instructions to change the parameters values are shown. Then run the MATLAB® program named Paramfcnen.m to get the state and input matrices of the desired system as programmed in the last "txt" file. The locations of the resistances are programmed randomly, but these can be modified. If the order and applied power of the actuators must be changed, modify since the line 90 of that program for the location of the actuators:

```
% Actuator vector (Order:  Nx (Rod), Nx*Ny (Plate), Nx*Ny*Nz (Cube))
% Command randint puts zero or one in a random way
qs1 = randint(100,1,[0,1]);
Qs = qs1;
```

Notice that the command *randint* assigns randomly ones or zeros to the vector "`qs1`" which has 100 elements or nodes. Modify this vector as its desired, i.e:

```
qs1 = [zeros(1,90),ones(1,10)];
```

turn off the first 90 resistances and turn on the last 10.

Modify the lines 162, 290, and 566 (for the rod, the plate, and the cube respectively) to apply the desired power to the resistances:

```
% Percentage of maximal power applied
Act = fact*qs_max*Qs;
```

where "`fact`" is a power percentage predefined in the .txt file.

Finally, run the dynamical system using the Simulink® program *CCModelSF.mdl*. This Simulink® code runs a S-Function named S_Tcont.m. The results can be viewed graphically running GraphicsCC.m.

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