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THEME – ICT
[Information and Communication Technologies]

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Table of contents

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Executive Summary

This deliverable describes the new proposed benchmarks cases and the exercises in the period of time M19-M36. During the first eighteen months of the project four different benchmark cases were proposed:

- a heat system
- a four tank system,
- an electric power system
- a chemical benchmark case

These benchmark cases are described in Deliverable D6.3.1 and main results during these eighteen months are described in deliverable D6.4.1. Also, models and additional information can be found in the Virtual Portal.
During the last eighteen months work has been focused on three benchmark cases: The first one is the four-tank system. A set of Distributed MPC algorithms has been tested and the results have been exhaustively analyzed and compared, also with a centralized MPC approach and a decentralized MPC algorithm.
Two new benchmark cases were proposed related to the HD-MPC applications of Workpackage 7:

- an irrigation canal system
- a hydropower valley system

These new benchmark cases and the main results are described in this document.
All the documentation about the four tank system and the hydropower valley benchmarks have been published in the public HD-MPC website, being accessible to the control community, including a description of the benchmark and experiments, models (linear and non-linear), etc.
The main results of the benchmark process have been published in journals and conferences:

- Journal of Process Control
- American Control Conference
- IFAC World Congress

Chapter 1

Synopsis

The second chapter presents the results on the four-tank benchmark. This benchmark is a real four-tank plant located in the Department of Ingeniería de Sistemas y Automática of the University of Seville. This benchmark has been used to compare the behavior of the different distributed MPC approaches developed in the scope of the HD-MPC Project.

The evaluation and comparison between the different controllers have been performed according to the following indices;

- Controller properties
  1. Modeling requirements: the class of models considered by each of the controllers, for instance linear/nonlinear, plant model or subsystem model, etc.
  2. Controller objectives: the properties addressed by the tested controllers, for instance optimality, constraint satisfaction, stabilizing design, recursive feasibility, etc.
  3. Auxiliary software needed: optimization routines, simulation routines, etc.
- Performance evaluation
  1. Performance index J: a measure of the performance of the controlled plant.
  2. Performance index during the transient J_t: a measure of the performance during the transient to remove the effect of steady offset.
  3. Settling time: a measure of the velocity of the controlled plant calculated by summing the settling times (defined as 95% achievement) after all steps in the reference.
  4. Number of floating point reals transmitted between the controllers per iteration.
  5. Number of data packets transmitted during a sampling time.

Seven different approaches were tested and compared, including two centralized MPC and a decentralized MPC. The tested algorithms were the following:

- Centralized MPC for tracking
- Centralized standard MPC for regulation
- Decentralized MPC for tracking

- Distributed MPC based on a cooperative game
- Sensitivity-Driven Distributed Model Predictive Control
- Feasible-cooperation distributed model predictive controller based on bargaining game theory concepts
- Serial DMPC scheme

The last four ones are distributed MPC algorithms developed by HD-MPC Consortium.

These controllers were based on different models and assumptions and provide a broad view of the different distributed MPC schemes developed within the HD-MPC project. The results obtained show how distributed strategies can improve the results obtained by decentralized strategies using the information shared by the controllers.

This work has been published in the Journal of Process Control (vol. 21, n.5, June 2011) and it is in the 4º place in the list of the most downloaded papers of the Journal in the period April-June 2011. (`www.elsevier.com/locate/jprocont`)

The third chapter introduces the Hydro power valley model.The system is a hydro power plant composed by several subsystems connected together. It is composed by 3 lakes and a river which is divided in 6 reaches which terminate with dams equipped with turbines for power production. The lakes and the river reaches are connected in three different ways: by a duct , ducts equipped with a turbine and ducts equipped with a pump and a turbine. The river is fed by the an upstream inflows and tributary flows. The models of the different component of the system and the proposed subsystem decomposition is presented in the chapter.

Two test scenarios are been considered:

- in the first scenario the power output of the system should follow a given reference while keeping the water levels in the lakes and at the dams as constant as possible;
- in the second scenario the system profit should be maximized based on the available information on the hourly electricity price variations.

The next chapter presents the Irrigation Canal Benchmark. The system to be controlled is an open-canal used for water distribution (for irrigation and supply of drinking water), composed of several reaches connected by gates with some reservoirs to store water and for regulation purposes.

The target is to control the management of water in order to guarantee flows requested by users (mainly irrigation districts). For this purpose, there are off-take gates at both sides of the canal, where water is taken from the canals for irrigation. The level of the canals must be maintained over a minimum value needed to take water in the off-take points along the channels.

The benchmark is a section of the ”postrasvase Tajo-Segura” in Spain. The selected section, described later, is a Y-shape canal, a main canal that splits into two canals with a gate placed at the input of each one of them . The length of the canals ares:

- Canal de la Pedrera,the total length of this canal is 6.680 km.
- Canal de Cartagena, in our case-study only a part of this canal is used (17.444km).

The total length of the canals is approximately 24 km.

The target is to control the management of water in canals in order to guarantee flows requested by users. For this purpose, it is necessary to maintain the level of the canal over the off-take gate when flow is requested.

The manipulated variables are the flow at the head of the canal and (if downstream control) and the position of the gates. There is a constraint on the flow at the head: The total amount of water over a determined time period is limited.

Main objective of the system is to satisfy user requests (if not there will be also a penalization) satisfying the constraints.

Regarding the Irrigation Canal Models, the dynamics of water flowing along open irrigation canals can be obtained by applying the Saint Venant equation. Nevertheless, because these equations are very complex to be used directly for control, simplified models usually linearized around a working point are used. The integrator-delay model, a first-order systems with a delay, are normally used to model the canal dynamics. Also, the connection between reaches is modeled using the gate discharge equation, a semi-empirical non-linear equation.

A hierarchical and distributed model predictive control approach applied to irrigation canal planning from the point of view of risk mitigation has been developed in HD-MPC Project. These approach has been applied successfully to the Irrigation Canal Benchmark.

The algorithm presents two levels in optimization. At the lower level, a distributed model predictive controller optimizes the operation by manipulating flows and gate openings in order to follow the water level set-points. The higher level implements a risk management strategy based on the execution of mitigation actions if risk occurrences are expected. Decision variables are mitigation actions which reduce risk impacts that may affect the system.

This work has been published in the Journal of Process Control (vol. 21, n.5, June 2011)

The final chapter is dedicated to dissemination of benchmark activities. The dissemination activities related to WP6 consist mainly in the publication of documentation in the public HD-MPC website, the use of the Virtual Portal (for consortium internal dissemination and the publication of some papers in Journals and Conference proceedings describing results of any of the benchmark.

Chapter 2

Four-tank system

2.1 The four tank plant and proposed experiment

The four-tank plant is a laboratory plant located in the Department of Ingeniería de Sistemas y Automática of the University of Seville that has been designed to test control techniques using industrial instrumentation and control systems. The plant consists of a hydraulic process of four interconnected tanks inspired by the educational quadruple-tank process proposed by Johansson [3]. A complete description of the plant and the models can be found in Deliverables D4.3.1 and D4.4.1 and in [2].

A continuous-time state-space model of the quadruple-tank process system can be derived from first principles to result in

$$\begin{aligned}
\frac{dh_1}{dt} &= -\frac{a_1}{S}\sqrt{2gh_1} + \frac{a_3}{S}\sqrt{2gh_3} + \frac{\gamma_a}{S}q_a, \\
\frac{dh_2}{dt} &= -\frac{a_2}{S}\sqrt{2gh_2} + \frac{a_4}{S}\sqrt{2gh_4} + \frac{\gamma_b}{S}q_b, \\
\frac{dh_3}{dt} &= -\frac{a_3}{S}\sqrt{2gh_3} + \frac{(1-\gamma_b)}{S}q_b, \\
\frac{dh_4}{dt} &= -\frac{a_4}{S}\sqrt{2gh_4} + \frac{(1-\gamma_a)}{S}q_a,
\end{aligned} \tag{2.1}$$

where h_i, S and a_i with $i \in \{1,2,3,4\}$ refer to the level, cross section and the discharge constant of tank i, respectively; q_j and γ_j with $j \in \{a,b\}$ denote the flow and the ratio of the three-way valve of pump j, respectively and g is the gravitational acceleration.

The following experiment is defined in which the control objective is to follow a set of reference changes in the levels of tanks 1 and 2, h_1 and h_2, by manipulating the inlet flows q_a and q_b based on the measured levels of the four tanks:

- The first set-points are set to $s_1 = s_2 = 0.65$ m. These are aimed to steer the plant to the operating point and guarantee identical initial conditions for each controller. Once the plant reaches the operating point the benchmark starts maintaining the operation point for 300 seconds.
- In the first step, the set-points are changed to $s_1 = s_2 = 0.3$ m for 3000 seconds.
- Then, the set-points are changed to $s_1 = 0.5$ m and $s_2 = 0.75$ m for 3000 seconds.
- Finally, the set-points are changed to $s_1 = 0.9$ m and $s_2 = 0.75$ m for another 3000 seconds. To perform this change tanks 3 and 4 have to be emptied and filled respectively.

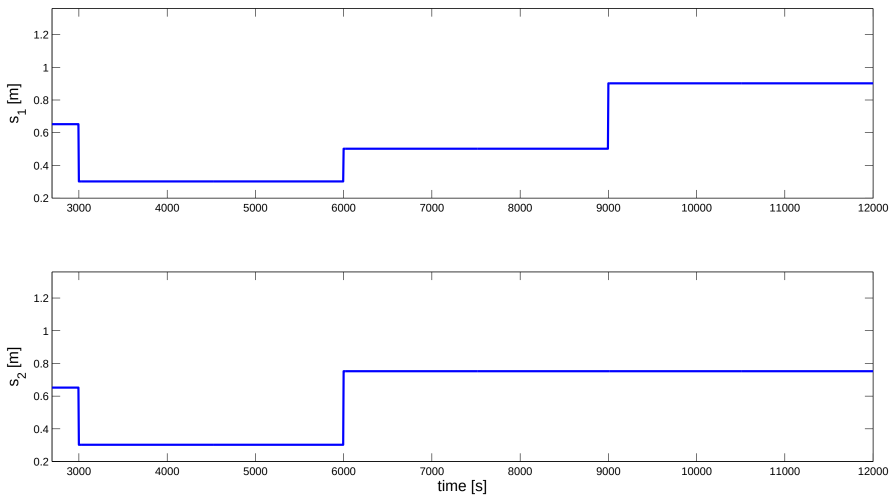

Figure 2.1: Set-point signals for the benchmark

The set-point signals are shown in Figure 2.1. The control test duration is 3 hours and 20 minutes. It is important to remark that the set-points have been chosen in such a way that large changes in the different equilibrium points are involved. This is illustrated in Figure 2.2, where the region of admissible sets points is depicted together with the proposed set-points. Notice that some of them are close to the physical limits of the plant in terms of inputs or level of the tanks 3 and 4.

The objective of the benchmark is to design distributed MPC controllers to optimize the performance index

$$J = \sum_{i=0}^{N_{sim}-1} (h_1(i) - s_1(i))^2 + (h_2(i) - s_2(i))^2 + 0.01(q_a(i) - q_a^s(i))^2 + 0.01(q_b(i) - q_b^s(i))^2$$

where q_a^s and q_b^s are the steady manipulable variables of the plant for the set-points s_1 and s_2 calculated from steady conditions of the proposed model of the plant. The tested controllers have been designed using a sampling time of 5 seconds. The performance index measures the response of the plant once it has been steered to the operation point. Then J is calculated during the time period $[2700, 12000]$ seconds, that is, for a total of $N_{sim} = 1860$ samples.

The four-tank benchmark is appropriate to compare the closed-loop performance of the different distributed MPC controllers considered because the two subsystems considered are highly coupled. However, other aspects such as the network communication and timing issues of the controllers are not apparent due to the small number of subsystems and the way the controllers are implemented. Nevertheless, these aspects can be studied by evaluating the data communication requirements between controllers. Then, the evaluation and comparison between the different controllers will be performed according to a collection of suitable indices, which are described in the following:

- Controller properties

  1. Modeling requirements: the class of models considered by each of the controllers, for instance linear/nonlinear, plant model or subsystem model, etc.
  2. Controller objectives: the properties addressed by the tested controllers, for instance optimality, constraint satisfaction, stabilizing design, recursive feasibility, etc.

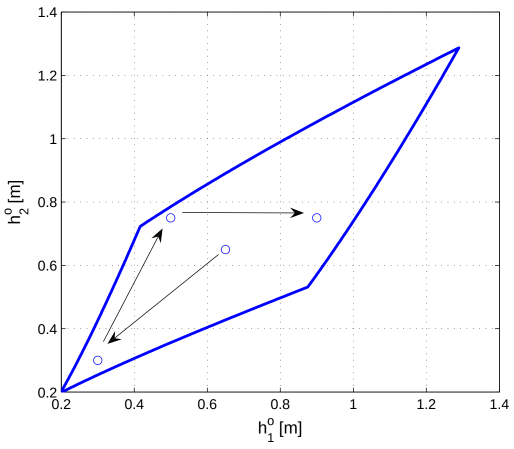

Figure 2.2: Set of admissible set-points.

3. Auxiliary software needed: optimization routines, simulation routines, etc.

- Performance evaluation

1. Performance index J: a measure of the performance of the controlled plant.
2. Performance index during the transient J_t: a measure of the performance during the transient to remove the effect of steady offset.
3. Settling time: a measure of the velocity of the controlled plant calculated by summing the settling times (defined as 95% achievement) after all steps in the reference.
4. Number of floating point reals transmitted between the controllers per iteration.
5. Number of data packets transmitted during a sampling time.

2.2 Tested predictive controllers

To design and tune the DMPC (Distributed Model Predictive Control) controllers, all the proposed algorithms used the same Simulink model of the nonlinear continuous time system which was identified at the University of Seville. Each controller has been implemented as a Simulink block and integrated in a Simulink control model similar to the simulation model used in the design stage. This Simulink control model communicates with the PLC of the real plant via the OPC protocol to receive the measured level of the tanks and to send the calculated manipulated variables.

In the following subsections, a brief description of the different control techniques are presented together with the results of the control test in the real plant. A complete description can be found in [1].

2.2.1 Centralized MPC for tracking

A centralized predictive controller based on the linearized prediction discrete-time model has been tested on the plant. Since the reference is changed throughout the control test, the MPC for tracking proposed in [4] has been chosen. This controller is capable to steer the plant to any admissible set point ensuring constraint satisfaction.

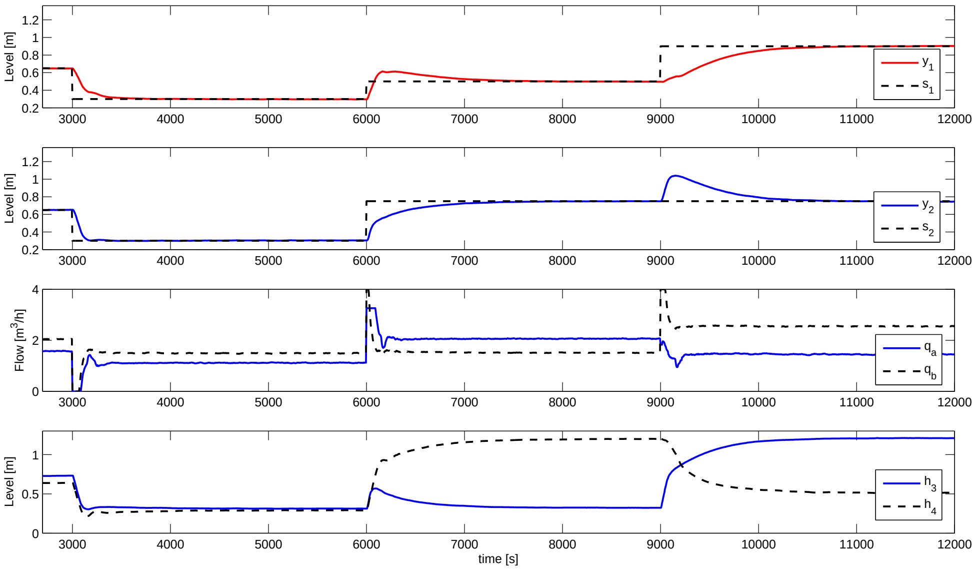

Figure 2.3: Evaluation of the control test in the real plant of the centralized MPC for tracking

This controller has been successfully tested on the real plant and the results are shown in Figure 2.3. The performance index for this test is $J = 28.4091$.

The MPC for tracking may exhibit a possible optimality loss due to the addition of the artificial reference as a decision variable. However, the standard MPC for regulation enjoys the local optimality property. In order to enhance the performance of the test, the standard regulation MPC has been applied and tested on the plant. Figure 2.4 shows the results obtained. The performance index for this test is $J = 25.4655$, hence better than the performance of the MPC for tracking. It is important to remark that, this controller does not guarantee feasibility, stability or constraint satisfaction when the set-point is changed, although for this particular case, these have been achieved.

2.2.2 Decentralized MPC for tracking

The second control technique tested has been a decentralized predictive controller. The considered subsystems have been chosen according to the pairings derived from relative gain array (RGA) analysis. The RGA matrix calculated for the linearized model results in

$$RGA = \begin{bmatrix} -0.4 & 1.38 \\ 1.38 & -0.4 \end{bmatrix}.$$

Considering the values of the RGA, it is decided to control the output h_1 with q_b, that is to control y_1 with u_2 in the subsystem 1, and h_2 with q_a, namely to control y_2 with u_1 in the subsystem 2.

A MPC for tracking has been designed for each subsystem.

The results of the experiments can be seen in Figure 2.5 for which the performance index is $J = 39.5421$.

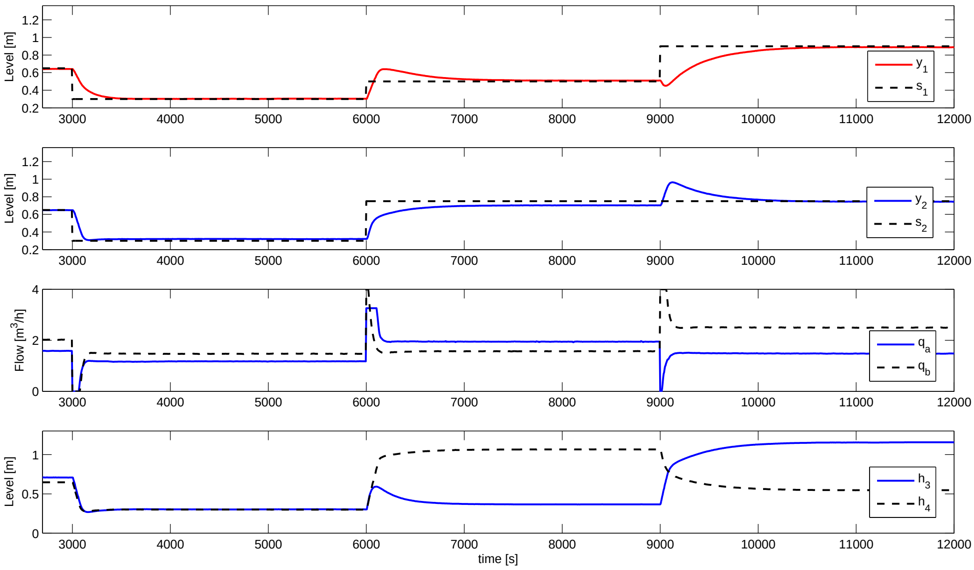

Figure 2.4: Evaluation of the control test in the real plant of the centralized MPC for regulation

2.2.3 Distributed MPC based on a cooperative game

The next algorithm is a distributed MPC scheme based on a cooperative game scheme presented in [5]. This control scheme considers a class of distributed linear systems where two subsystems coupled with the neighboring subsystem through the inputs are defined by

$$\begin{aligned} x_1(k+1) &= A_1 x_1(k) + B_{11} u_1(k) + B_{12} u_2(k), \\ x_2(k+1) &= A_2 x_2(k) + B_{21} u_1(k) + B_{22} u_2(k), \end{aligned} \tag{2.2}$$

where $x_i \in \mathbb{R}^{n_i}$, $i = 1,2$, are the states of each subsystem, $u_i \in \mathbb{R}^{m_i}$, $i = 1,2$, are the different inputs and $A_1, A_2, B_{11}, \ldots$ are matrices of appropriate dimensions.

The control objective is to regulate the system to the set points while guaranteeing that a given set of state and input constraints are satisfied. The proposed distributed scheme assumes that for each subsystem, there is a controller that has access to the model and the state of that subsystem. The controllers do not have any knowledge of the dynamics of their neighbor, but can communicate freely among them in order to reach an agreement on the value of the inputs applied to the system. The proposed strategy is based on negotiation between the controllers on behalf of a global performance index. At each sampling time, agents make proposals to improve an initial feasible solution on behalf of their local cost function, state and model. This initial feasible solution is obtained from the optimal solution of the previous time step. These proposals are accepted if the global cost improves the corresponding cost of the current solution.

The MPC controllers minimize the sum of two local performance indexes J_1 and J_2 that depend on the future evolution of both states and inputs. Each controller solves a sequence of reduced dimension optimization problems to determine the future input trajectories U_1 and U_2 based on the model of its subsystem. Details of the algorithm proposed can be found in [5].

In order to test the proposed DMPC scheme a discrete-time linear model around the equilibrium point h_0, q_0 (which corresponds to the first reference) has been obtained linearizing the nonlinear

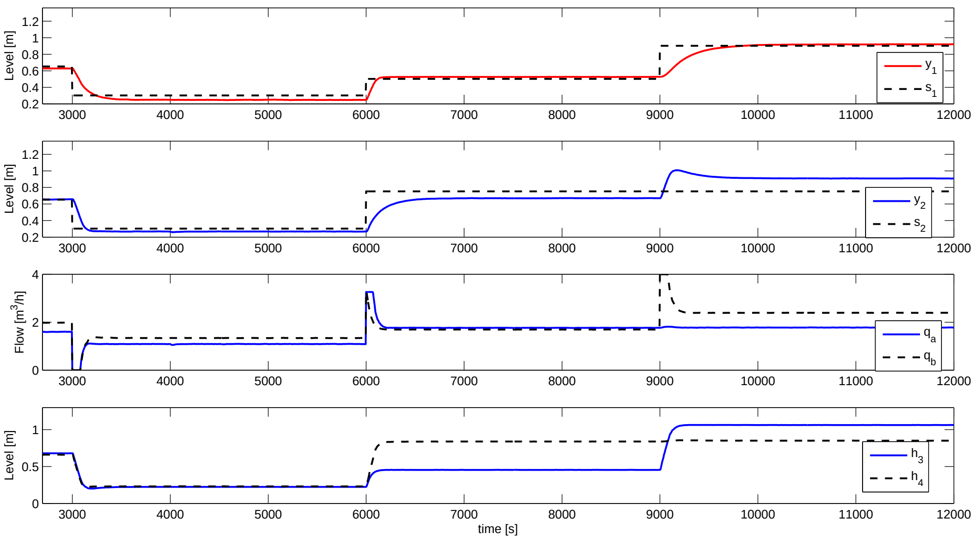

Figure 2.5: Evaluation of the control test in the real plant of the decentralized MPC

model of the quadruple-tank process with a sampling time of 5 sec.

The objective of the MPC controllers is to minimize a performance index that depends on the future evolution of both states and inputs based on the following local cost functions

$$
\begin{aligned}
J_1(x_1,U_1,U_2) &= \sum_{j=1}^{N}(x_{1,j}-x_{1r})^T Q_1 (x_{1,j}-x_{1r}) + \sum_{j=0}^{N-1} R_1(u_{1,j}-u_{1r})^2,\\
J_2(x_2,U_2,U_1) &= \sum_{j=1}^{N}(x_{2,j}-x_{2r})^T Q_2 (x_{2,j}-x_{2r}) + \sum_{j=0}^{N-1} R_2(u_{2,j}-u_{2r})^2,
\end{aligned}
$$

where $N=5$, $x_{i,j}$ and $u_{i,j}$ are the j-steps ahead predicted states and inputs of controller i respectively. The variables $x_{i,r}$ and $u_{i,r}$ are the target state and input obtained from the difference between the equilibrium point and the reference levels and flows. To determine these values, the nonlinear model has been used to obtain the levels of h_3, h_4 and the corresponding equilibrium flows q_a, q_b that guarantee that the references are an equilibrium point of the system. This implies that it has been done in a centralized manner. The controllers receive the appropriate references as inputs.

The weighting matrices were chosen to minimize the benchmark objective function, that is, $Q_1 = Q_2 = I$, $R_1 = R_2 = 0.01$. The local controller gains for each controller were $K_1 = (0.17, 0.21)$ and $K_2 = (-0.16, -0.14)$. These gains were designed with LMI techniques based on the full model of the system in order to stabilize both subsystems independently while assuring the stability of the centralized system. The role of these gains is important because the option in the game that allows to guarantee closed-loop stability is constructed shifting the last decided control action; that is, the first element is dropped after it is applied to the system and a term evaluated with these gains is added at the end of the horizon control vector (see [5] for more details).

The designed controller has been successfully tested on the real plant; the trajectories are shown in Figure 2.6. The performance index of the test is $J = 29.5787$. The performance index is close to the performance index of the centralized MPC for regulation. Note however that the input trajectories are not smooth because the controllers switch between different modes.

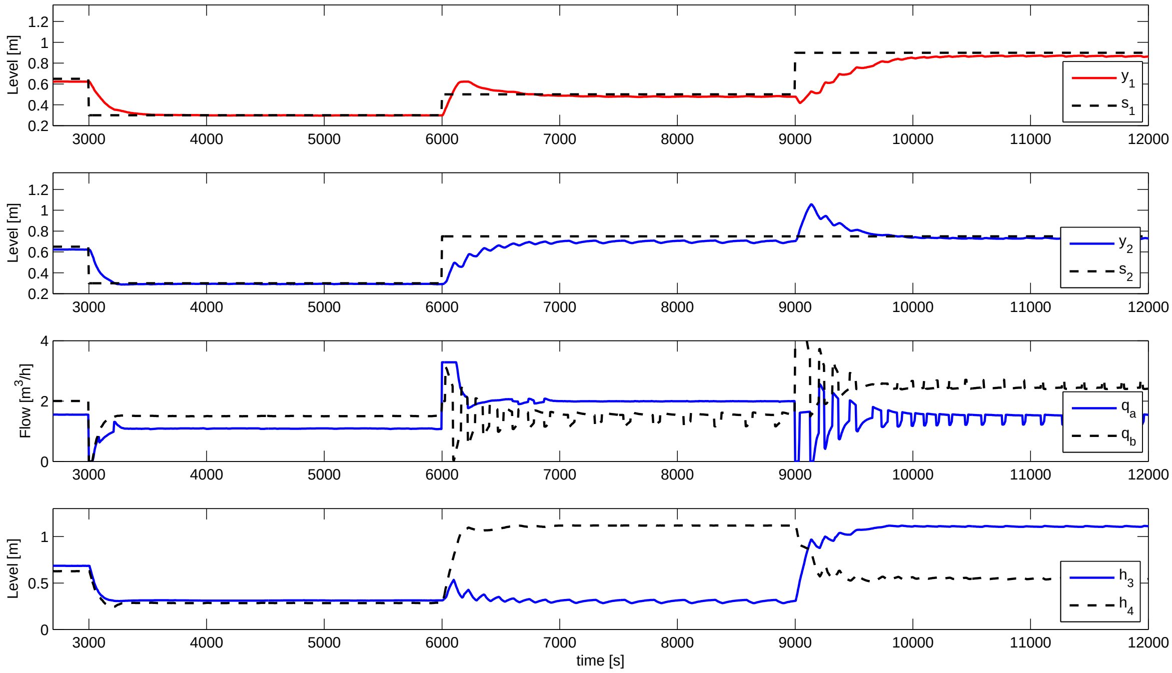

Figure 2.6: Evaluation of the control test in the real plant of the DMPC based on a cooperative game

2.2.4 Sensitivity-driven distributed model predictive control

A sensitivity-driven distributed model predictive control (SD-DMPC) scheme [9] is considered in this subsection. SD-DMPC is based on a new distributed dynamic optimization method employing a sensitivity-based coordination mechanism [8]. For the distributed controllers the four-tank system is decomposed first using an RGA analysis.

The method is implemented with a prediction horizon of $N = 100$ (500 sec.) in order to achieve a stable closed-loop control. The input variables u_i have been discretized using 3 parameters for each input. One parameter has been chosen to reflect the steady state values, while the others have been chosen to approximate the transient part within the first 10 seconds of the horizon by piece-wise constant representations.

We have tested the controller for three different configurations:

(a) With a fixed number of 3 iterations, i.e. an implementation without convergence leading to sub-optimal control,

(b) with a fixed number of 10 iterations for optimal control, and

(c) with a fixed number of 10 iterations and an additional Kalman filter to handle steady state disturbances.

Due to the strong coupling of the subsystems, convergence of the method is rather slow. It is possible to achieve optimality in approximately 10 iterations. However, already with only three iterations, good performance can be achieved. The performance index in the real plant for the configurations

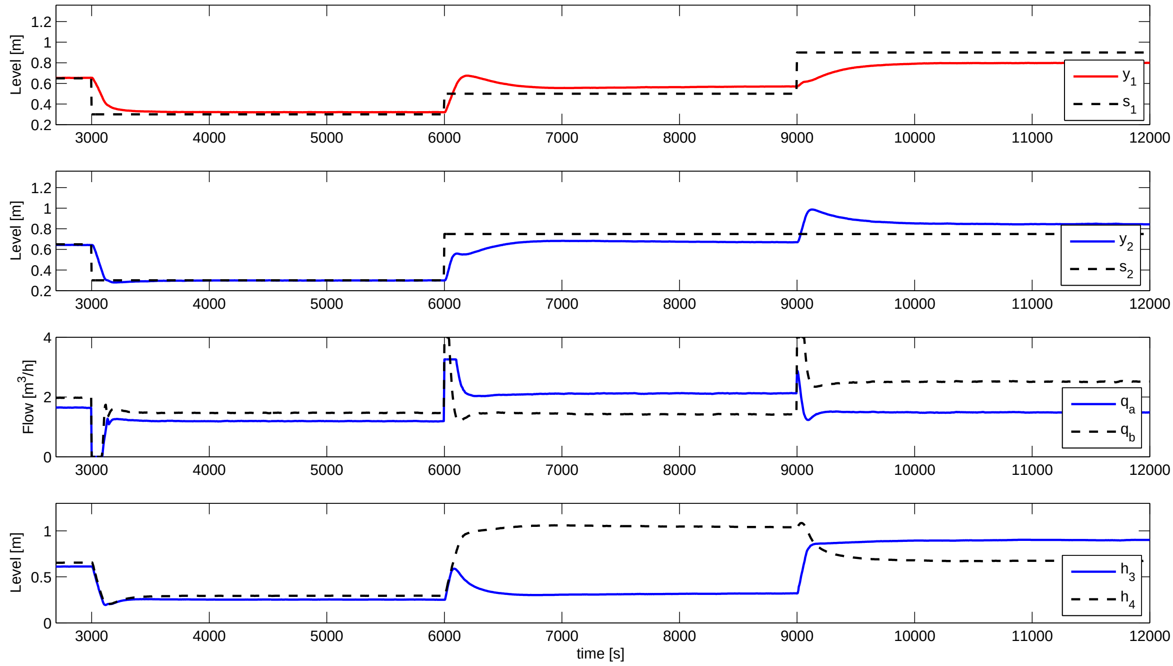

Figure 2.7: Evaluation of the control test in the real plant of the SD-DMPC

investigated are $J = 45.072$ for configuration (a), $J = 35.525$ for configuration (b), and $J = 28.616$ for configuration (c). The trajectories for configuration (b) are shown in Figure 2.7, while the trajectories for configuration (c) are given in Figure 2.8. The Kalman filter in configuration (c) is able to estimate the steady state disturbances d_i of the plant successfully, such that the steady state control errors vanish. A nonsmooth behavior of the controlled flow rates q_a and q_b can be observed, which is induced by the Kalman filter and could be reduced by a better tuning of the filter. So far, the controllers have only been tuned in a simulation environment and applied to the real plant without further tuning.

2.2.5 Feasible-cooperation distributed model predictive controller based on bargaining game theory concepts

The next applied algorithm is a Feasible-cooperation distributed model predictive controller based on bargaining game theory concepts. From the point of view of game theory, DMPC is a game in which the players are the subsystems, the actions are the control inputs, and the payoff of each subsystem is given by the value of its cost function.

In the case of the four-tank plant, the whole system model has been decomposed into two subsystems as described previously The prediction models used are

$$\begin{aligned} x_i(k+1) &= A_i x_i(k) + B_i u(k) \\ y_i(k) &= C_i x_i(k), \qquad i = 1,2, \end{aligned} \tag{2.3}$$

where $x_1(k) = [h_1(k), h_3(k)]^T$, $x_2(k) = [h_2(k), h_4(k)]^T$, $u(k) = [q_1(k), q_2(k)]^T$, and A_i, B_i, C_i, i, $i = 1,2$, are submatrices of the system A, B, C of the discrete-time linear model of the four-tank system.

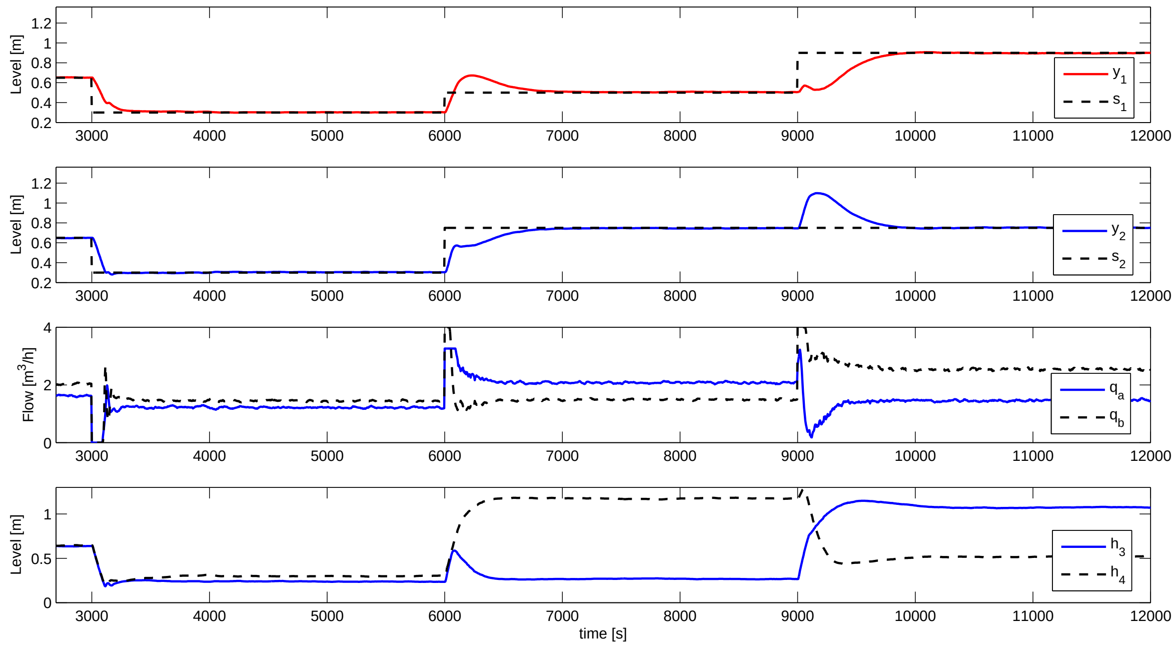

Figure 2.8: Evaluation of the control test in the real plant of the SD-DMPC with Kalman filter

Figure 2.9 shows the behavior of the four-tank system, when the DMPC controller based on game-theoretical concepts computes the optimal control inputs. The performance index calculated for the control test is $J = 46.3177$.

Note that the aim of the game-theoretical formulation of the DMPC problem is that the subsystems cooperate while obtaining some benefit. From Figure 2.9, it is possible to conclude that this aim is achieved, because the pumps are working jointly in order to reach the reference values for the levels h_1 and h_2, which is the global control objective. Also, the control decisions are taken in a cooperative way. Therefore, when the changes in the reference values were applied, the pumps react with the purpose of achieving the new operation point in a cooperative fashion without sacrificing the local performance.

2.2.6 Serial DMPC scheme

We have also implemented the scheme proposed in [6, 7] for the four-tank system. This scheme is derived from a serial decomposition of an augmented Lagrangian formulation of the centralized overall MPC problem. This results in a scheme in which controllers perform at each control step a number of iterations to obtain agreement on which actions should be taken. The goal of the iterations is to obtain actions that are optimal from a system-wide point of view using only local models and measurements and communicating only with neighboring agents on values of interconnecting variables.

In the particular case of the four-tank system two subsystems are defined, similarly as in Section 3.2.2. The first subsystem has as state $x_1 = [h_1, h_3]$, input $u_1 = q_b$, and output $y_1 = h_1$; the second subsystem has as state $x_2 = [h_2, h_4]$, input $u_2 = q_a$, and output $y_2 = h_2$. Furthermore, $w_{\text{in},21} = q_a$,

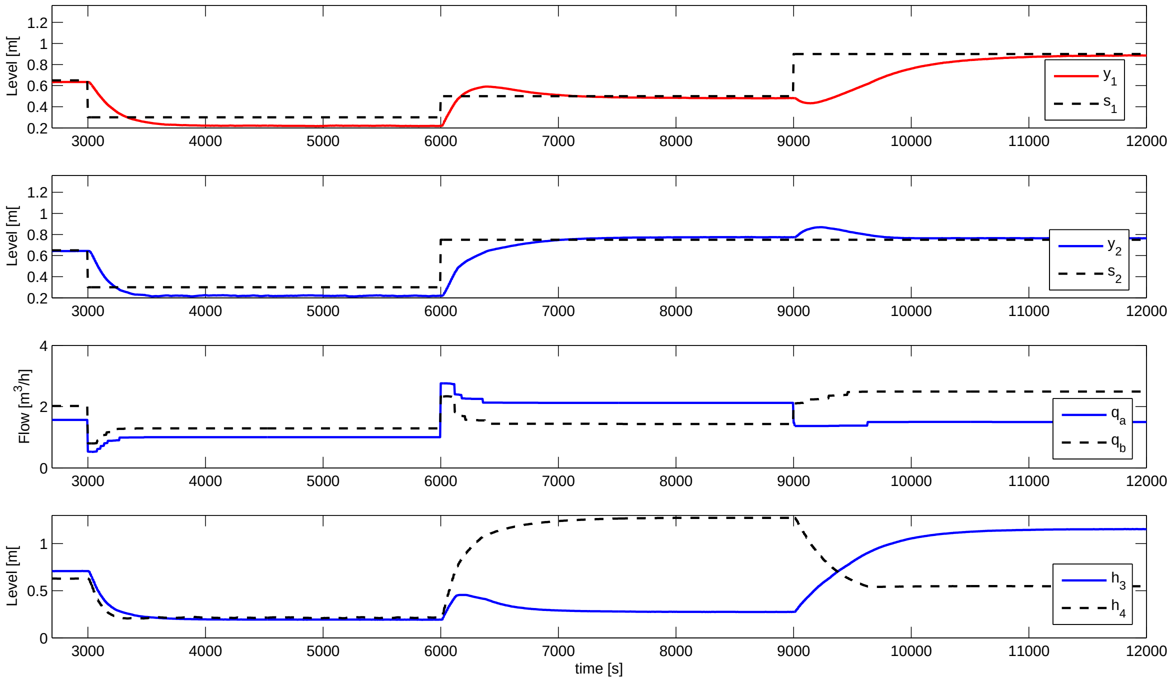

Figure 2.9: Evaluation of the control test in the real plant of the distributed model predictive controller based on a bargaining game

$w_{\text{out},12} = q_a$, $w_{\text{in},12} = q_b$, $w_{\text{out},21} = q_b$. The local objectives are defined as follows:

$$J_{\text{local},1} = \sum_{l=0}^{N-1} \left((h_1(k+1+l) - s_1(k+1+l))^2 + 0.01(q_b(k+l) - q_b^s(k+l))^2 \right)$$
$$J_{\text{local},2} = \sum_{l=0}^{N-1} \left((h_2(k+1+l) - s_2(k+1+l))^2 + 0.01(q_a(k+l) - q_a^s(k+l))^2 \right).$$

Moreover, the subsystem matrices that define the dynamics are obtained easily from the discretized overall system matrices.

The control test of the proposed controller performed in the four-tank plant is shown in Figure 2.10. The calculated performance index for this controller is $J = 44.60$.

2.2.7 Comparative results

The two tables presented in this section, summarized the main results of the different tested approaches.

Table 2.1 shows some qualitative properties of these controllers. The entry *Model Requirements* shows whether the controllers need full or partial knowledge of the system and whether the model used is linear or nonlinear. The entry *Control Objectives* shows whether the controller is optimal from a centralized point of view (i.e., provides the same solution as the centralized MPC for regulation), guarantees constraint satisfaction if a feasible solution is obtained and whether it can be designed to

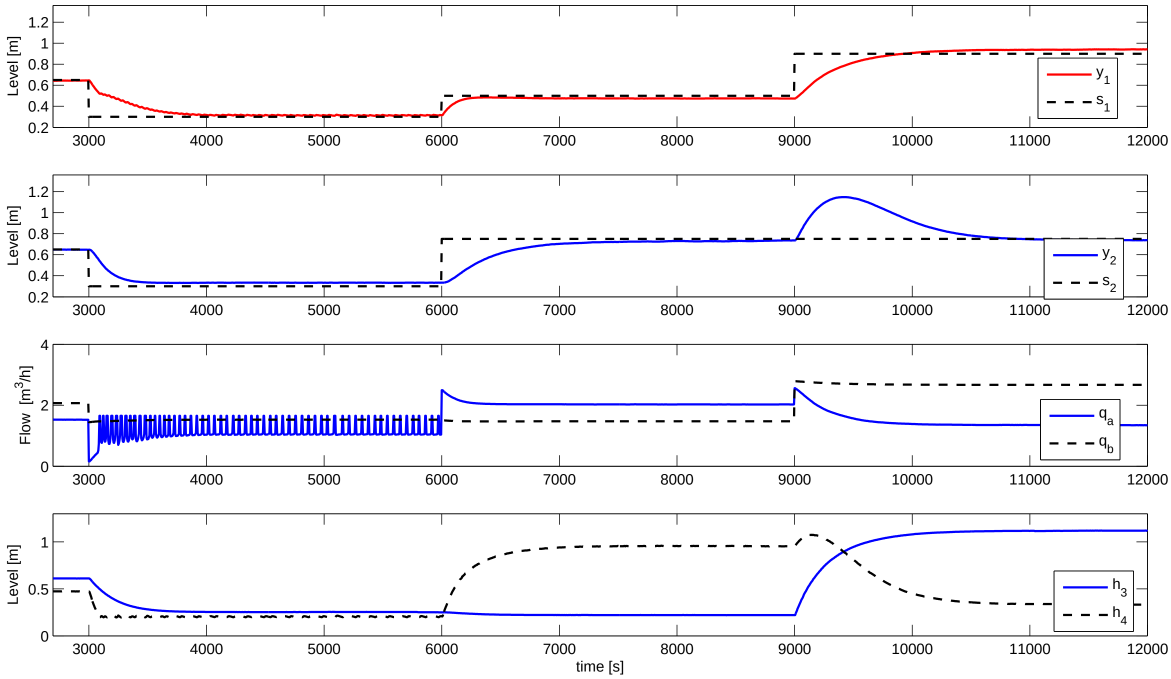

Figure 2.10: Evaluation of the control test in the real plant of the Serial DMPC

guarantee closed-loop stability in a regulation problem. The *Auxiliary Software* entry shows which type of additional software is needed by each controller of the distributed scheme.

Finally, some quantitative results are presented in Table 2.2

Qualitative properties	Model Requirements	Control Objectives	Auxiliary Software
Centralized Tracking MPC	Linear system Full model	Suboptimal Constraints Stability	QP
Centralized Regulation MPC	Linear system Full model	Optimal Constraints Stability	QP
Decentralized MPC	Linear system Local model	Suboptimal	QP
DMPC Cooperative game	Linear system Local model (*Full model*)	Suboptimal Constraints (*Stability*)	QP
SD-DMPC	Linear system Local model	Optimal Constraints	QP
DMPC Bargaining game	Linear system Local model	Suboptimal Constraints	NLP
Serial DMPC	Linear system Local model	Optimal Constraints	QP

Table 2.1: Table of qualitative properties of each tested controller.

Control performance	J	J_t	t_s	N	# floats	# trans
Centralized Tracking MPC	28.4	28.12	3280	5	N.D	N.D.
Centralized Regulation MPC	25.46	23.78	2735	5	N.D	N.D.
Decentralized MPC	39.54	21.2	1685	5	0	0
DMPC Cooperative game	30.71	28.19	2410	5	20	3
SD-DMPC (w/o KF)	35.65	23.28	2505	100	33	10
SD-DMPC (with KF)	28.61	28.26	1895	100	33	10
DMPC Bargaining game	46.32	39.52	3715	5	6	2
Serial DMPC	44.59	41.94	3130	5	10	[2,7]

Table 2.2: Table of the quantitative benchmark indexes of each tested controller

Chapter 3

Hydropower valley benchmark

3.1 Description of the system

3.1.1 System overview

The system we consider is a hydro power plant composed by several subsystems connected together. Figure 3.1 gives an overview of the system which is composed by 3 lakes (L_1, L_2 and L_3) and a river which is divided in 6 reaches (R_1, R_2,R_3, R_4, R_5 and R_6) which terminate with dams equipped with turbines for power production (D_1, D_2,D_3, D_4, D_5 and D_6). The lakes and the river reaches are connected by a duct (U_1), ducts equipped with a turbine (T_1 and T_2) and ducts equipped with a pump and a turbine (C_1 and C_2). The river is fed by the flows q_{in} and $q_{tributary}$.

In the following sections we shall provide a model for all the subsystems. To simplify the system modeling we make the following assumptions:

- the ducts are connected at the bottom of the lakes (or to the bottom of the river bed);
- the cross section of the reaches and of the lakes is rectangular;
- the width of the reaches varies linearly along the reaches;
- the river bed slope is constant along every reach.

3.1.2 System model

Reach model

The model of the reaches is based on the one-dimensional Saint Venant partial differential equation:

$$\frac{\partial q(t,z)}{\partial z} + \frac{\partial s(t,z)}{\partial t} = q_l(t) \tag{3.1}$$

$$\frac{1}{g}\frac{\partial}{\partial t}\left(\frac{q(t,z)}{s(t,z)}\right) + \frac{1}{2g}\frac{\partial}{\partial z}\left(\frac{q^2(t,z)}{s^2(t,z)}\right) + \frac{\partial h(t,z)}{\partial z} + I_f(t,z) - I_0(z) = 0 \tag{3.2}$$

The two above equations express the mass and momentum balance. The variables represent the following quantities:

- $q_l(z)$ is the lateral in flow per space unit;

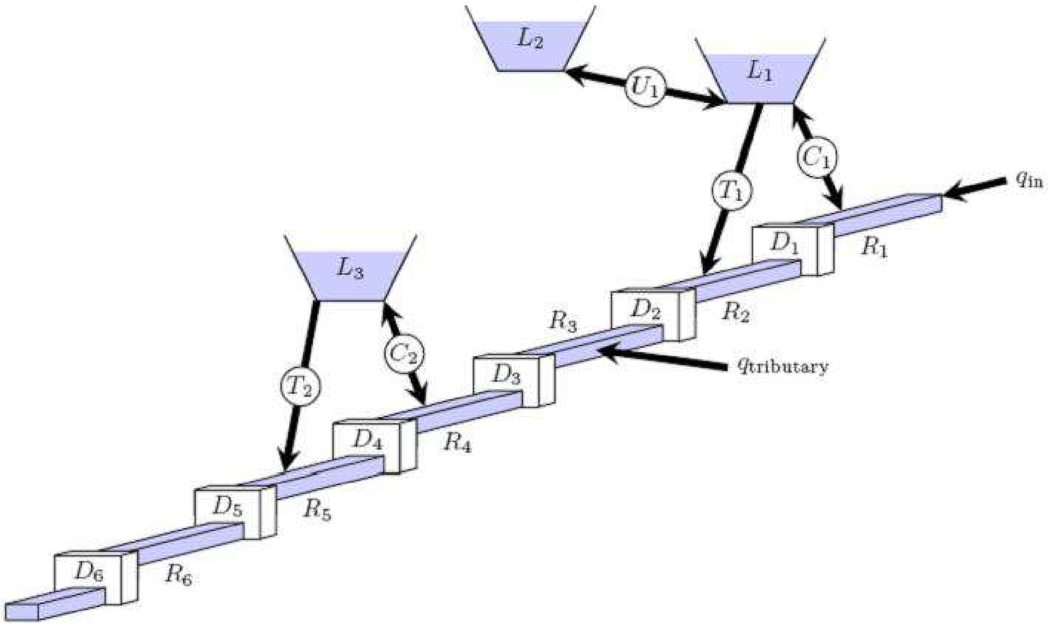

Figure 3.1: Overview of the Hydro power valley.

- z is the spatial variable which increases along the flow main direction;
- $q(t,z)$ is the river flow (or discharge) at time t and space coordinate z;
- $s(t,z)$ is the wetted surface;
- $h(t,z)$ is the water level w.r.t. the river bed;
- g is the gravitational acceleration;
- $I_f(t,z)$ is the friction slope;
- $I_0(z)$ is the river bed slope.

Assuming the cross section of the river is rectangular we can write the following equations:

$$s(t,z) = w(z)h(t,z) \tag{3.3}$$

and

$$I_f(t,z) = \frac{q(t,z)^2(w(z)+2h(t,z))^{4/3}}{k_{\text{str}}^2(w(z)h(t,z))^{10/3}} \tag{3.4}$$

The partial differential equation can be converted into an ordinary differential equation with the method of lines by dividing the reach into N cells of length dz.

Lake model

Denote by $q_{in}(t)$ and $q_{out}(t)$ the water inflow and outflow of the lake under consideration. The volume of water inside the lake varies accordingly to the following equation

$$\frac{dh(t)}{dt} = \frac{q_{in}(t) - q_{out}(t)}{S} \tag{3.5}$$

where $h(t)$ is the water level and S is the lake surface area.

Duct model

The flow inside the duct U_1 can be modeled using Bernoulli's law. Assuming the duct section is much smaller than the lake surfaces, the flow from lake L_1 to lake L_2 can be expressed as

$$q_{U_1}(t) = S_{U_1}\, sign(h_{L_2}(t) - h_{L_1}(t) + \Delta h_{U_1})\sqrt{2g\,|h_{L_2}(t) - h_{L_1}(t) + \Delta h_{U_1}|} \tag{3.6}$$

where h_{L_1} and h_{L_2} are the water levels for lakes L1 and L2, Δh_{U_1} is the height difference of the duct, $S_{U_1}1$ is the section of the duct and g is the gravitational acceleration.

Turbine model

For every turbine we assume we can control directly the turbine discharge. The power produced is given by the following equation

$$p_t(t) = k_t q_t(t)\Delta h_t(t) \tag{3.7}$$

where k_t is the turbine coefficient, $q_t(t)$ is the turbine discharge and $\Delta h_t(t)$ is the turbine head.

Pump model

Pumps can be modeled similarly to turbines. The power absorbed by a pump is given by

$$p_p(t) = k_p q_p(t)\Delta h_p(t) \tag{3.8}$$

where k_p is the pump coefficient, $q_p(t))$ is the pump discharge and $\Delta h_p(t))$ is the pump head.

Subsystem partition

The system is partitioned into 8 subsystems in the following way:

- Subsystem 1 is composed by lakes L_1 and $L2$ and ducts U_1, T_1 and C_1. Duct C_1 can function as a pump or a turbine.
- Subsystem 2 is composed by lake L_3 and ducts T_2 and C_2.
- Subsystems 3, 4, 5, 6, 7, and 8 are composed by a reach and dam. Figure 3.2 represents the structure of the dams.All the flow going through the dams is used by the turbine to produce electricity. The head of the turbines inside the dams can be expressed as difference of the water level before and after the dam. Since the water level after dam D_6 is not part of the model we consider it constant

There are also constraints of maximum and minimum values in the level of the lakes and the reaches and in flows in ducts and dams.

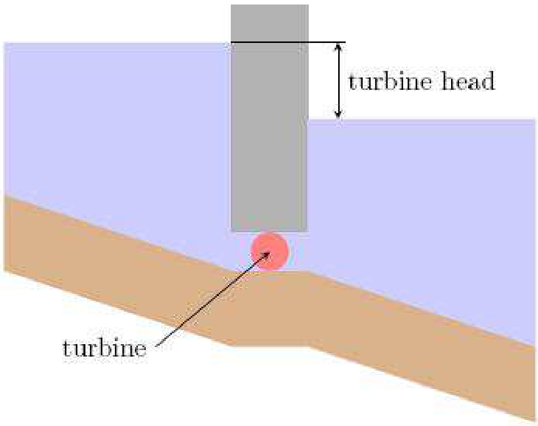

Figure 3.2: Dam configuration.

Control test scenarios

We consider two test scenarios:

- in the first scenario the power output of the system should follow a given reference while keeping the water levels in the lakes and at the dams as constant as possible;
- in the second scenario the system profit should be maximized based on the available information on the hourly electricity price variations.

To simplify the description of the two optimal control problem formulations we define

- $x_i(t)$: state vector of subsystem i;
- $u_i(t)$: input vector of subsystem i;
- C_i: set describing the constraints for subsystem i;
- $p_i(x_i(t), u_i(t))$: power produced by subsystem i;

First Scenario: Power reference tracking

We assume that the power reference to be followed by the entire system is known 24 hours in advance. Therefore,the prediction horizon is set to 86400 seconds. The inputs of the system can be changed every 30 minutes.Accordingly, the input vectors $u_i(t)$ will be piecewise constant. In order to achieve a good tracking performance while keeping the water levels of the lakes and at the dams as constant as possible, the cost of the optimal control is composed by two terms. The first term is a penalty on the power tracking error. The second term is a penalty on the deviation of the water level of interest w.r.t. a reference value. The cost function to be minimize is

$$\min_{x_i,u_i} \quad \int_0^{86400} \gamma \left| p_r(t) - \sum_{i=1}^{8} p_i(x_i(t), u_i(t)) \right| dt + \sum_{i=1}^{8} \int_0^{86400} (x_i(t) - x_{ss,i})^T Q_i (x_i(t) - x_{ss,i}) dt \quad (3.9)$$

The function $p_r(t)$ is the given power reference (piecewise constant). $x_{ss,i}$ is a given steady state for the system which is used as a reference for the water levels.

Second Scenario: Profit maximization

When maximizing the profit of the plant the electricity price is known 24 hours in advance and varies every hour. As in the power reference tracking scenario the inputs can be modified every 30 minutes.

The proposed cost function is

$$\min_{x_i,u_i} \quad \sum_{k=0}^{24} c_k \int_{k3600}^{(k+1)3600} \sum_{i=1}^{8} p_i(x_i(t),u_i(t))dt + \sum_{i=1}^{8} c_{f,i}^T x_i(T) \tag{3.10}$$

where c_k is the electricity price (in EURO/W) during the k-th hour, and $c_{f,i}$ is a vector pricing the water remaining in the system at the end of the control horizon.

3.2 DMPC algorithms

The following approaches are being applied to the proposed benchmark by the HD-MPC consortium.

- Approximate subgradient method.
- Hierarchical MPC controller with RTO (Real-Time Optimizer) coordinator
- Distributed MPC controller with RTO coordinator
- S-DMPC - linear quadratic constrained optimal
- Multiple shooting for distributed system
- Distributed model predictive control based on a cooperative game

The results of these DMPC controllers will be reported in Deliverable D7.1.3 (Report that presents the closed-loop validation results for the combined cycle start-up and for the hydro-power valley, including stability and constraints issues), *as well as the HD-MPC demonstration of results*.

Chapter 4

Irrigation canal benchmark

4.1 Description of the system

4.1.1 Introduction

The system to be controlled is an open-canal used for water distribution (for irrigation and supply of drinking water), composed of several reaches connected by gates with some reservoirs to store water and for regulation purposes.

The target is to control the management of water in order to guarantee flows requested by users (mainly irrigation districts). For this purpose, there are off-take gates at both sides of the canal, where water is taken from the canals for irrigation. The level of the canals must be maintained over a minimum value needed to take water in the off-take points along the channels

Nowadays, the management of the canal is as follows:

The institution in charge of supplying the water and managing the main canals is the Regional Government. There is an annual planning of flows to supply to users during the whole year and in addition to this, every week, irrigation districts make a forecast of their demand (flow). Attending to this request and taking into account the weather prediction, the Regional Government fixes the flow upstream at the head of the installation and manipulates main gates.

These take-off gates are manipulated by the irrigation districts according with their needs, but with the hard constraint of the annual water volume, and trying to follow their planned flows. So, it can be considered that there is available a prediction of these off-take flows.

Nowadays, most of the gates are operated manually, but there is a work in progress to install local controllers on the gates to control the level upstream. Also, all the information is going to be centralized in a Control Center, where will be possible to fix set-points and operate gates.

The benchmark is a section of the ”postrasvase Tajo-Segura” in Spain. The selected section, described later, is a Y-shape canal, a main canal that splits into two canals with a gate placed at the input of each one of them (4.1). The length of the canals ares:

- Canal de la Pedrera,the total length of this canal is 6.680 km.
- Canal de Cartagena, in our case-study only a part of this canal is used (17.444km).

The total length of the canals is approximately 24 km.

The most important elements in the canals are the main gates which regulate the level of water along the canals and the off-take gates, where farmers take water from the canals for irrigation. There are 7 main gates and 17 off-take gates in the section selected, hence, we have considered that the

Figure 4.1: Scheme of the benchmark canal

system is composed of seven subsystems. Each subsystem begins at one of the main gates and ends at the next one. There are, however, two exceptions. The subsystems that begin at the gates labeled CCMICAR-08 and CCMIPED-01 do not have a gate at the end because they are the last of each one of the branches.

The main characteristics of this type of installation are the following:

- The proposed benchmark is a small piece of a real canal, but typical figures of these installations are tens of gates and hundreds of off-take points distributed along a canal in a tree structure. Typical lengths are hundred of kilometers.
- The management of big canals usually is distributed. The canal is divided into several areas, controlled by different Control Centers. Sometimes, different areas are managed by different organizations.
- Non linear behavior. The most extended way to model water level and flows is with the Saint-Venant equations. These are non linear partial differential equations (a mass and a momentum balance).

The interest of the proposed benchmark can be summarized as follows:

- Distributed control: irrigation canals have been previously proposed as experimental plant to validate distributed MPC techniques in several papers. The main reason of this choice relies on the interconnected nature of the system. This interconnection can be seen from a physical point of view (reaches are connected through gates) but also from a functional point of view (the canal is divided into several areas, sometimes controlled even by different organizations).
- Hierarchical control: also irrigation canals have been previously used as a platform to test hierarchical control techniques. Usually, the control structure of the plant is oriented to a hierarchical control with the lowest control loop to regulate the flows or level upstream or downstream

the gates, then an upper level to test the advanced control technique, producing set-points. Finally, the higher level checks the safety of the plant and decides the optimal operating point of the plant.

- Perturbations take an important role in this system because off-take gates and pumps are not manipulated by the control systems. Sometimes these off-takes flows are measured, sometimes only an aggregate value of these flows is available and finally, also it is possible to have not measures of these perturbations. Usually there is an estimation of the flows (a weekly plan). Also rainfalls can be considered as perturbations.

4.1.2 Objectives

The target is to control the management of water in canals in order to guarantee flows requested by users. For this purpose, it is necessary to maintain the level of the canal over the off-take gate when flow is requested.

The manipulated variables are the flow at the head of the canal and (if downstream control) and the position of the gates. There is a constraint on the flow at the head: The total amount of water over a determined time period is limited.

Main objective of the system is to satisfy user requests (if not there will be also a penalization) satisfying the above constraints. Another objective to be considered is the minimization of the leaks and evaporation (function of the levels) and also to minimize maintenance costs (the maintenance of concrete blocks and junctions is better if they are submerged, so high levels are preferred for that purpose).

4.1.3 Variables

Controlled variables

Basically, there are two classical control strategies: In upstream control the controlled variable is the upstream level besides the gate, while in downstream control the controlled variable is the level at the end of the downstream reach (that is the level upstream the next gate in downstream direction).

The selection between downstream or upstream control depends mainly of how the canal is managed. Downstream control is more common in irrigation canal literature, but in Spain most of the canal are upstream controlled.

These levels have the physical limitations of maximum and minimum values.

Also, it is possible to use flows through gates as controlled variables. For example, in the bifurcation in an upstream control configuration, one of the gates is controlling flow (both gates cannot control the same level)

Manipulated variables

If a direct control is selected, then the manipulate variable is the opening of each gate. These signals are continuous and again they have maximum and minimum values.

Otherwise, if a two level control structure is chosen then the reference of each local flow control loop is considered as manipulated variable.

Disturbances

- Off-takes flows are usually measured, but as these gates are not operated by the canal manager, the measures are not always sent to the Canal Control Center. Sometimes, only an aggregate value of the off-takes of one or more reaches is available. Nevertheless, a prediction can be obtained from the weekly flows plan.
- Rainfalls: Sometimes this variable is measured and also it can be predicted by weather forecast models.

4.2 Irrigation canal models

The dynamics of water flowing along open irrigation canals can be obtained by applying the Saint Venant equations, described previously in Chapter 3.

$$\frac{\partial q(t,z)}{\partial z} + \frac{\partial s(t,z)}{\partial t} = q_l(t) \tag{4.1}$$

$$\frac{1}{g}\frac{\partial}{\partial t}\left(\frac{q(t,z)}{s(t,z)}\right) + \frac{1}{2g}\frac{\partial}{\partial z}\left(\frac{q^2(t,z)}{s^2(t,z)}\right) + \frac{\partial h(t,z)}{\partial z} + I_f(t,z) - I_0(z) = 0 \tag{4.2}$$

(See above eqs. 3.1 and 3.2 for the meaning of the terms)

Because these equations are very complex to be used directly for control, simplified models usually linearized around a working point are used. The integrator-delay model, a first-order systems plus a delay, are normally used to model the canal dynamics (see [7]).

Considering a typical irrigation canal divided into several reaches separated by gates. Let consider the downstream water levels $h_i(t)$ as controlled variables and the gate opening $u_i(t)$ as the manipulated variables. Each canal reach has an inflow from an upstream canal reach $Q_{in,i}$ and an out flow to a down stream canal reach $Q_{o,i}$. Furthermore, other flows are considered as perturbation variables:

- $q_{in,i}$, flows due to rainfall, failure in upstream gate.
- $q_{o,i}$, known off-take out flows by farmers,considered as measurable perturbations.

The discrete model considered using the defined variables is:

$$A_i(h_i(k+1) - h_i(k)) = T_d(Q_{in,i}(k-t_d) + q_{in,i}(k) - Q_{o,i}(k) - q_{o,i}(k)) \tag{4.3}$$

where T_d is the length of the sampling time, A_i the surface of the reach and t_d the delay of the input Q_{in} (the level is measured downstream). The discharge through a submerged flow gate can be determined by the expression

$$Q_o(t) = C_d L\sqrt{2g}u(t)\sqrt{h_{up}(t) - h_{dn}(t)} \tag{4.4}$$

where C_d is the gate discharge coefficient, L is the gate width, $u(t)$ the gate opening and $h_{up}(t)$, $h_{dn}(t)$ the upstream and downstream water levels,respectively.

4.3 A HD-MPC approach based on a risk mitigation perspective

A hierarchical and distributed model predictive control approach applied to irrigation canal planning from the point of view of risk mitigation has been developed in HD-MPC Project. These approach has been applied successfully to the Irrigation Canal Benchmark.

The algorithm presents two levels in optimization. At the lower level, a distributed model predictive controller optimizes the operation by manipulating flows and gate openings in order to follow the water level set-points. The approach described in [5], that also has been applied also to the four-tank benchmarks, has been used.

The higher level implements a risk management strategy based on the execution of mitigation actions if risk occurrences are expected. Risk factors such as unexpected changes in demand, failures in operation or maintenance costs are considered in the optimization. Decision variables are mitigation actions which reduce risk impacts that may affect the system. This work shows how model predictive control can be used as a decision tool which takes into account different types of risks affecting the operation of irrigation canals.

In the following, same results are presented:

4.3.1 Higher level

- Main target: to minimize cost due to internal and external risks and to determine the level set point that minimize risks
- There is a 365 day study period (1 year) and a 1 day sampling time.
- Prediction horizon, $N = 5$ days.
- Manipulated variables: mitigation actions.

Figure 4.2 shows the mitigation of two risks:

- Farmers water demand varies from forecast.
- Rainfall changes water level of canal, producing water logging of adjacent lands.

The mitigation action applied to these risks is the variation of the level set-point.

Figure 4.3 shows the effect of risk mitigation on operation costs. The cost is reduced when risk mitigation is considered.

4.3.2 Lower level

- Main target: to control water management in canals in order to guarantee flow demanded by users.For this purpose, it is necessary to maintain the level of the canal over the off-take gate when flow is requested.
- Controlled variables: upstream levels at the gates, h_i.
- Manipulated variables: flow at the head of the canal and the position of the gates
- Constraints: Maximum and minimum levels to guarantee that off-take points are submerged, maximum and minimum gates opening.

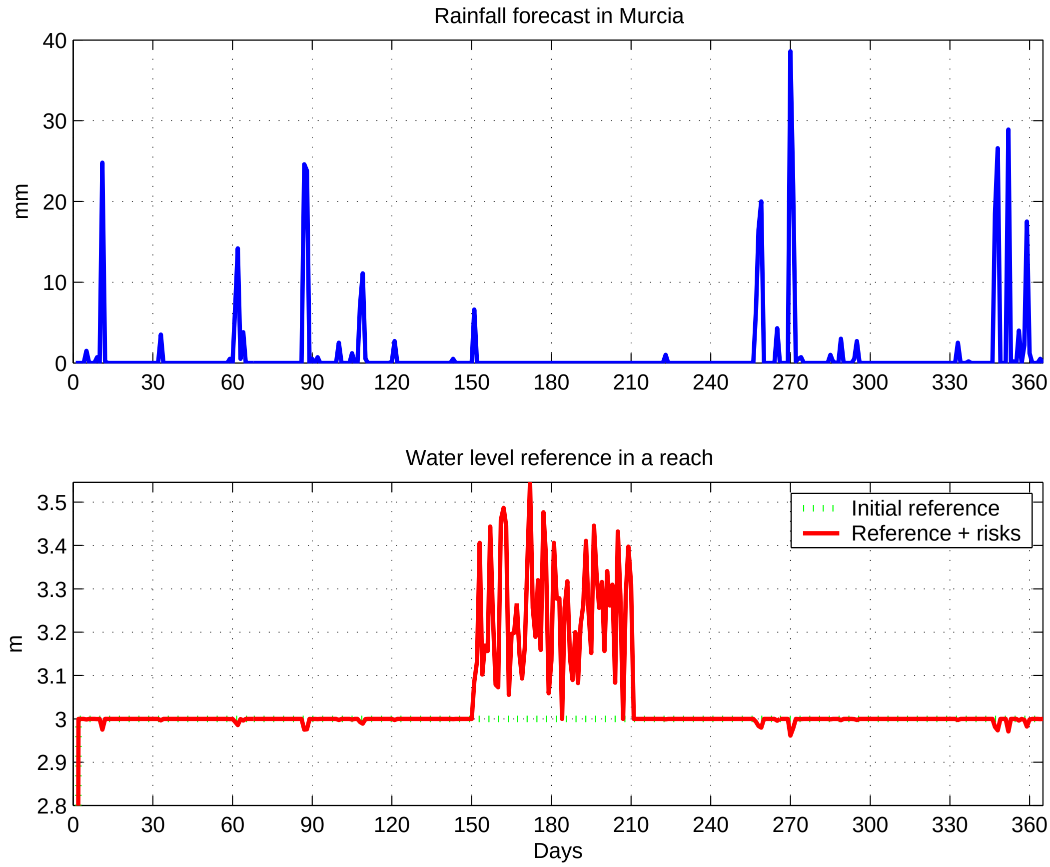

Figure 4.2: Top panel: rainfall forecast in Murcia. Lower panel: level reference in one reach by considering risks.

- The study period is 1440 min (1day) and the sampling time 1 min. This period corresponds to what happens during the day 150 of the higher controller period.
- The control horizon is set to $N_c = 5$ for all the agents. The prediction horizon for the agent i is equal to the control horizon plus the delay k_i of the reach, that is, $N_p(i) = N_c + k_i$.

In the presented scenario, all the reaches begin with a water level of 3.0 m. and there is a change of reference for all the reaches to 3.40 m. at time $k = 0$. This change is originated at the higher control level as a function of the risk mitigation policy. In particular, the change of reference corresponds to the day 150 in Figure 4.2, where the evolution of the references during a 1-year period is depicted. The simulation shown in Fig. 4.4 corresponds to the nominal case,that is, the simulation was performed without disturbances. It can be seen how the reference is followed for all the reaches.

A complete description of the algorithm and the application to the benchmark can be found in deliverable D7.3.3 and in [10].

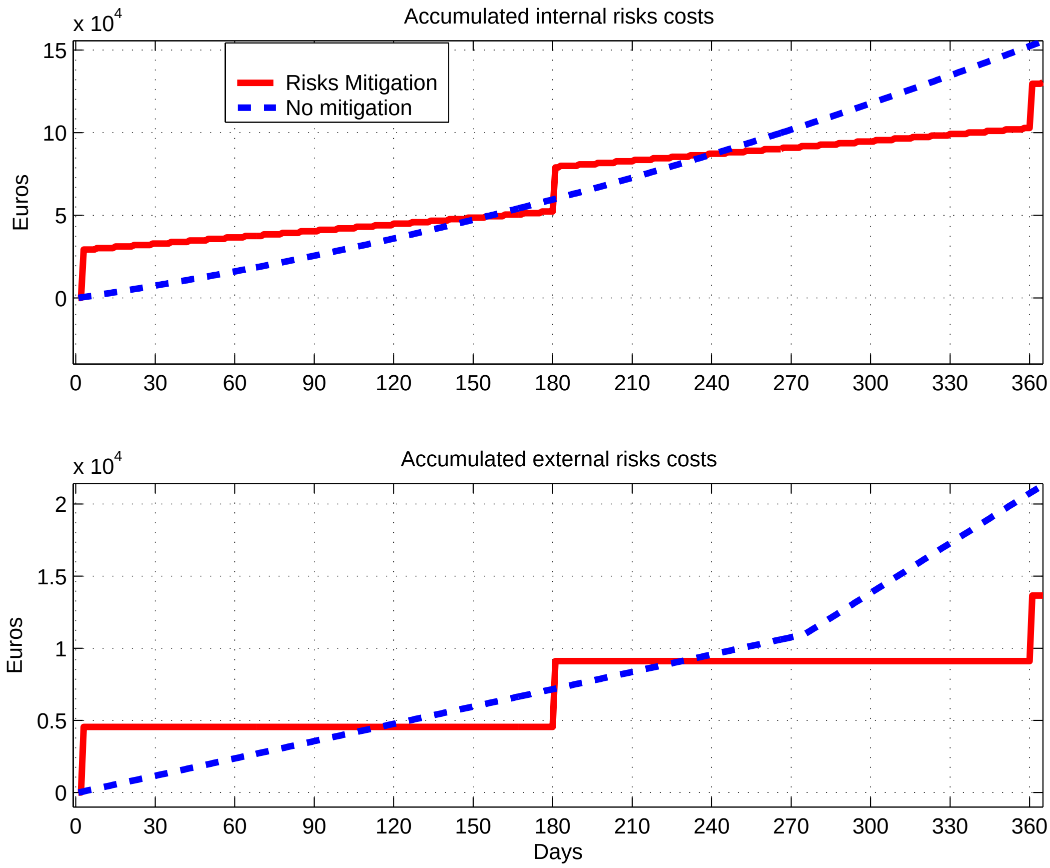

Figure 4.3: Optimization of the cost by considering risks.

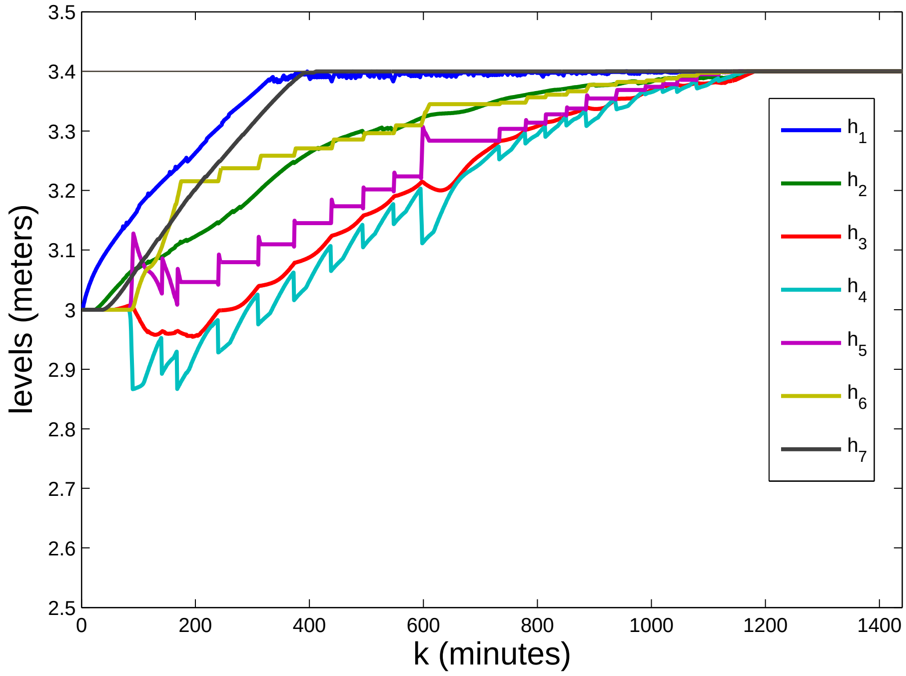

Figure 4.4: Levels in reaches for day 150 for the nominal case

Chapter 5

Dissemination

The dissemination activities related to WP6 consist mainly in the publication of documentation in the public HD-MPC website, the use of the Virtual Portal (for consortium internal dissemination and the publication of some papers in Journals and Conference proceedings describing results of any of the benchmark.

- HD-MPC website: `http://www.ict-hd-mpc.eu`
- Virtual Portal: `http://nyquist.us.es/hdmpcproject`

It is important to remark that the paper ”A comparative analysis of distributed MPC techniques applied to the HD-MPC four-tank benchmark”, published in the Journal of Process Control (vol. 21, n.5, June 2011) is in the 4º place in the list of the most downloaded papers of the Journal in the period April-June 2011. (`www.elsevier.com/locate/jprocont`)

5.1 Public Benchmarks

The consortium has defined to the benchmarks as public cases, in such a way that all the documentation is available to the control community to test their distributed approaches and compare them with the results of the HD-MPC consortium approaches. The two public benchmark cases are:

- Four-tank system (Simulated results)
- Hydropower valley

The following information has been uploaded and available in the public HD-MPC website related to each one of the benchmarks:

- Description of the system, including the proposed subsystem decomposition
- Objectives and description of the experiment, including a cost function
- Non-linear model to be used as a simulation model
- Linear model for linear MPC approaches
- Results of different approaches (centralized and decentralized MPC are available for the four tank systems), including experimental results and performance criteria.

5.2 Publications

The following publications are directly related to benchmark cases:

- Alvarado, D. Limon, D. Muñoz de la Peña, J.M. Maestre, M.A. Ridao, H. Sheu, W. Marquart, R.R. Negenborn, B. De Schutter, F. Valencia, and J. Espinosa. ”A comparative analysis of distributed MPC techniques applied to the HD-MPC four-tank benchmark”. Journal of Process Control. volume 21, issue 5, June 2011, pp. 800-815.
- C. Savorgnan, C. Romani, A. Kozma, and M. Diehl. ”Multiple shooting for distributed systems with applications in hydro electricity production”. Journal of Process Control. volume 21, issue 5, June 2011. pp. 738-745
- A. Zafra-Cabeza, J.M. Maestre, M.A. Ridao, E.F. Camacho, and L. Sánchez. ”A hierarchical distributed model predictive control approach to irrigation canals: A risk mitigation perspective”. Journal of Process Control. volume 21, issue 5, June 2011. pp. 787-799.
- A. Zafra-Cabeza, J.M. Maestre, M.A. Ridao, E.F. Camacho, and L. Sánchez. ”Hierarchical Distributed Model Predictive Control : An Irrigation Canal Case Study”. American Control Conference 2011. pp. 3172-3177.
- C. Savorgnan, A. Kozma, J. Andersson, and M. Diehl. ”Adjoint-Based Distributed Multiple Shooting for Large-Scale Systems”. Proceedings of the 18th IFAC World Congress, 2011.
- A. Ferramosca, D. Limon, J.B. Rawlings, and E.F. Camacho. ”Cooperative Distributed MPC for Tracking”. Proceedings of the 18th IFAC World Congress, 2011.

5.3 Internal Dissemination

The objective of the Virtual Portal is to permit the communication among HD-MPC partners and to share experiences, documentation and software in a virtual space. Also it serves as a document repository and distribution tool among all project participants, ensuring the privacy requirements of contents.

The Virtual Portal includes a section dedicated to Workpackage 6, where the consortium participant can find documentation, model guides, models, experiment description, results, etc., about the six benchmark cases used in the Project.

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